In quest of Olympiad problems

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In an earlier article [1], where the activities of the Problems Committee of the Australian Mathematical Olympiad Committee (AMOC) were summarised, readers were invited to donate problems for use in national, regional and international mathematics competitions. Perhaps another call for olympiad-type problems will serve as a reminder to all who have been aware of those contests and at the same time inform those who have joined our mathematical community more recently of a noble way of enhancing school mathematics for budding mathematicians.

Participants in those competitions are usually senior secondary-school students, although brilliant younger students have been identified through the mathematical olympiads almost every year. The problems they have to solve are from "pre-calculus" areas: number theory, geometry (with a strong preference for "Euclidean" geometry), algebra, discrete mathematics, inequalities, functional equations. In [1], a sample of five competition problems was reproduced to give the potential problem donor an idea what type of problems would be appropriate. It might be useful to repeat that exerise, this time with competition problems that were set during the last two years.

(1) The AMOC Senior Contest is held in August of each year. About 100 students, most of them in year 11, are confronted with five problems and are given four hours to solve them. The following problem, Question 1 of the 2001 contest, rated "of easy to medium difficulty":

Prove that there is no function f defined for all real numbers and taking real numbers as values such that for every real number r,

(i)
$$f(r^2) - (f(r))^2 \geq \frac{1}{4},$$

(ii) the equation $f(x) = r$ has at most one solution;

(2) Question 3 of the 2003 AMOC Senior Contest rated as "medium to hard":

For any three distinct real numbers x, y, z, let

$$E(x,y,z) = \frac{(|x|+|y|+|z|)^3}{|(x-y)(y-z)(z-x)|}.$$

Determine the minimum possible value of $E(x, y, z)$.

(3) The Australian Mathematical Olympiad (AMO) is a two-day event in February with about 100 participants. On either day, students are given a four-hour paper containing four problems. The following problem, Question 7 of the 2002 AMO, turned out to be of medium difficulty:

Let n and q be integers, $n \geq 5$, $2 \leq q \leq n$. Prove that $q-1$ divides $\lfloor \frac{(n-1)!}{q} \rfloor$.

[Note: $\lfloor x \rfloor$ is the largest integer not exceeding x.]

(4) Here is a medium-to-hard geometry problem from the 2003 AMO (Question 6):

Let AD be a median of triangle ABC. Let point E lie on AD (extended if necessary) such that CE is perpendicular to AD. Suppose that angle ACE equals angle ABC.

Prove that either $AB = AC$ or BAC is a right angle.

(5) The Asian Pacific Mathematics Olympiad (APMO) takes place in March. The contest is a four-hour event with five problems to be solved. About twenty countries, most of them from the Pacific Rim, take part in the APMO. Usually, 20 to 25 Australian students are invited to participate in this competition. Here is a Question 1 from the 2003 APMO, which Australian students found of "easy to medium" difficulty:

Let a, b, c, d, e, f be real numbers so that the polynomial

$$p(x) = x^8 - 4x^7 + 7x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

factorises into eight linear factors $x - x_i$, with $x_i > 0$ for $i = 1, 2, \ldots, 8$. Determine all possible values of f.

Problems of recent International Mathematical Olympiads have been reported regularly in the *Gazette*. The complete set of AMO problems and solutions covering the period 1979–1995 can be looked up in [2], whereas the problems and solutions of all APMOs between 1989 and 2000 have appeared in [3].

Problem donations will be gratefully received by me as Chair of the AMOC Problems Committee.

References

[1] H. Lausch, *A call for olympiad problems*, Aust. Math. Soc. Gazette **28** (2001), 143–145.

[2] H. Lausch and P. Taylor, *Australian Mathematical Olympiads 1979–1995*, Australian Mathematics Trust, Canberra 1997.

[3] H. Lausch and C. Bosch Giral, *Asian Pacific Mathematics Olympiads 1989–2000*, Australian Mathematics Trust, Canberra 2000.

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