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Gazette

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- Reviews of books, particularly by Australian authors, or books of wide interest
- Classroom notes on presenting mathematics in an elegant way
- Items relevant to mathematics education
- Letters on relevant topical issues
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Deadlines for submissions to Volumes 34(3) and 34(4) of the *Gazette* are 1 June 2007 and 1 August 2007.

Volume 34 Number 2 2007

Editorial

Welcome to the second issue of the *Gazette* in 2007.

As most of our readers would be aware, the media have picked up the crisis that the mathematical sciences face in Australia. However, despite their interest in the National Strategic Review of Mathematical Sciences Research in Australia, the Government has not reacted and the decimation of mathematical sciences departments continues. Gia Underwood, NTEU representative at RMIT, wrote to us to point out the situation at RMIT, asking for support from the mathematical community. The international mathematical and statistical community is standing by their Australian colleagues. You can read the Open Letter sent to the Prime Minister, prepared by a sub-group of the Review Working Party, urging the Government to take action in response to the Review. The letter was signed by more than 100 international and nearly 400 Australian mathematicians and statisticians.

There are also opportunities for Australian mathematicians to provide support for the international community. Kevin Burrage reports on his time at the African Institute of Mathematical Sciences, an initiative that aims to contribute to the development of Africa by providing their graduates access to excellent mathematics training. It is an inspiring project and mathematicians from around the world can become involved.

This issue features the first of what we hope will be many interesting and thought-provoking contributions in the new column Classroom Notes. We hope the column will present innovative and elegant approaches to teaching mathematics, and invite contributions from all our readers. In our first Classroom Notes, Steve Sugden describes how he uses spreadsheets to teach mathematics, and how this tool provides an avenue for teaching classes of mixed mathematical levels. In an era where students may bring very few mathematical skills to their university study, innovative approaches such as Steve's are urgently needed. Many of our readers will identify with Steve's description of his students' skills and his frustration. The aim of the Classroom Notes is to share experiences and successful approaches in teaching mathematics. If you think your current teaching practice is novel and of interest to others, we would like to hear about it.

Also inside this issue are our regular contributions including the Style Files from Tony Roberts, and the second Puzzle Corner from Norman Do. We are sure that Norman's first Puzzle Corner (from last issue) got you thinking, and this issue's Corner is sure to give you more to mull over. But don't forget to send in your solutions; there is a $50 book token on offer for best submission. The deadline for submission for solutions to Puzzle Corner 1 (last issue) is fast approacing on 1 May, while solutions to Puzzle Corner 2 (from this issue) must be in by 1 July. See page 75 for more information.

Happy reading.

Birgit, Rachel and Eileen

President's column

Peter Hall*

The RQF: Concentrating on our research strengths

By now, most members of the Society working in Australian universities will have been involved to some extent in plans for the Research Quality Framework. Some members have expressed concern about the views voiced by management in their universities, and have raised apprehensions about the influence the RQF might have.

We seem destined to have some sort of RQF in the long-term future. While the federal opposition has declared itself to be opposed to the currently planned RQF, and indicated that, if elected, it would introduce an alternative that was less expensive to implement and concentrated more on measures of quality, Labor's RQF would still have some of the key characteristics of the Government's proposal.

In particular, both the Government and the opposition envisage a research assessment procedure that encourages Australian universities to concentrate on their research strengths. We all know that when a government says to universities that it is going to 'encourage' them to do something, it means that it will reorganise the distribution of funding so as to make doing anything else particularly distasteful and unattractive.

In an environment where research funding is not increasing, concentrating on our research strengths means discarding those aspects of our research where we are not so strong. Despite the difficulties we are experiencing today, in the mathematical sciences in Australia we still enjoy very significant research strengths, and it is feasible to enhance them at the expense of research in other parts of the mathematical sciences. However (and in this I'm sure I differ from many Australian university managers), it is not clear to me that we should be setting about enhancing all our strengths and, correspondingly, winding down the other areas where we don't have quite so much of a reputation.

The UK Research Assessment Exercise (RAE), on which the RQF is loosely based, had in part this effect. It was at least partially responsible for a decline in UK expertise in strategically important areas such as modern analysis, some parts of PDE, and statistics, as universities withdrew from those fields where they had perceived weaknesses and directed funding elsewhere. Of course, relatively small fields were the most vulnerable. If a research area where the UK was not widely represented became temporarily 'weak' in a particular university, perhaps because of a retirement or resignation, it was unlikely to be brought up to strength quickly because of the difficulty of finding suitable staff close at hand. As a result the research area became vulnerable to predatory strategic planning for future RAEs.

Thus, there are dangers in restricting attention to our strengths, and narrowing diversity in order to increase funding in areas where we perform best. This is particularly true in a small country like Australia, capable of fielding only modest research resources. Australia

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needs both depth and breadth in the mathematical sciences, both now and in the future. Shedding breadth for the sake of depth is not necessarily a wise decision, especially in the 21st century when the mathematical sciences are becoming increasingly multidisciplinary. Multidisciplinarity is present both within mathematics, where tools from several areas are used to solve problems in one particular area; and between mathematics and many different fields of science and engineering, where a wide variety of mathematical methods are used to solve problems of substantial, immediate practical importance.

How can we preserve diversity in scientific research, and at the same time enhance our strengths, in an environment where research funding is at best static? Perhaps the only feasible approach is to have periodic reviews of research in Australian universities, not just in the mathematical sciences but also in other fields, conducted largely by scientists and scholars based abroad. Those reviews should identify gaps in our capability that need to be filled, as well as areas where we are excellent. They should be authoritative, and have teeth; that is, our research funders and research managers must heed the reviewers' advice.

Other aspects of the issue of focusing on our research strengths should also give us cause to reconsider the directions we are taking. Over the last decade Australian universities have developed a significant redundancy culture, where staff with continuing appointments are dismissed or come under pressure to leave their jobs. The ardour of universities for enhancing their research strengths, by discarding continuing staff who might not augment the institutions' RQF performance, is currently causing significant stress in some Australian universities.

In this respect the Australian RQF experience is likely to differ substantially from its counterpart for the UK RAE. Australian university managers are more enthusiastic about making academic staff redundant, or threatening to make them redundant, than are their counterparts in any other country of which I'm aware. Quite apart from the highly unproductive tensions that waves of redundancies introduce to the workplace, they create obstacles to hiring strong research staff from abroad.

Let me give an example. I was overseas in April last year when *The Australian* published an article on the efforts being made at a Group of Eight university to shed staff who, the university's managers felt, might not enhance that institution's RQF performance. The URL[1] for the article zipped around the world within hours of appearing on the web. It motivated discussion, within the Australian expatriate academic community and among foreign scientists and scholars, of the nature of workplace relations in Australian universities and of the undesirability of taking an academic position there. Although this discussion initially centred on the particular institution involved, it quickly broadened to a critique of management capabilities in all our universities. I expect that recent media attention given to the current round of RQF-motivated redundancies will have provoked similar discussion abroad.

For all these reasons, moves in our universities to focus more sharply on Australia's research strengths, and eliminate weaknesses, need to be made more carefully and thoughtfully than they have been in the past. Australian university managers should take pains to ensure that they enhance, rather than damage, the nation's long-term research performance.

[1]The original article is no longer available free of charge on the web, but a copy can be accessed through HighBeam Research at http://calbears.findarticles.com/p/articles/mi_hb4692/is_200604/ai_n17518181

Letter to the editors

RMIT University staffing cutbacks

Early this year RMIT University announced staffing cutbacks within the mathematics and statistics discipline in their School of Mathematical and Geospatial Sciences. The University is planning to reduce its mathematics & statistics academic staff numbers by 25% by the end of the year. In February, three academic staff, with over 80 years teaching experience between them, were told that they were targeted for redundancies. More redundancies are expected in the second half of this year. Some staff in this discipline have already left the University and some vacancies will not be filled.

The University has told the NTEU that the cutbacks are due to a budget shortfall. Apparently the shortfall has occurred due to a decrease in student numbers in Schools that the discipline services, such as Engineering and Applied Sciences, and an increase in the level of financial return expected from the School to the University. This financial return, calculated from Teaching & Learning and commercial activity within the School, has increased from 43.5% in 2005 to 56.1% this year.

It is not surprising the School is struggling, and the NTEU have asked both the Vice Chancellor, Professor Margaret Gardner and the Pro-Vice Chancellor of the Science, Engineering & Technology portfolio, Professor Daine Alcorn, to support the School and halt the redundancies — not only in the interests of the individual staff members but in the national interest. The request has been denied by both.

Aside from the obvious threat to staff member job security, this erosion of the mathematics skill base at RMIT is of deep concern. Maths is a cornerstone for all technological studies. When maths-based subjects are not taught by properly qualified mathematicians, quality can only suffer as a result. As a member of the Australian Technology Network Group, this not only puts RMIT's reputation as a quality provider in the science and engineering fields at risk, it also accelerates the downward spiral towards the crisis Australia is facing.

We have established a petition on our website, which we are asking people to put their name to. The petition, addressed to the VC, requests that all redundancies are halted. This is not only in the interests of the affected staff, but in the national interest in keeping existing academics within this field in Australia. The Web address is http://www.nteu.org.au/bd/rmit .

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Classroom notes

Spreadsheets: an overlooked technology for mathematics education

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Electronic spreadsheets have been with us for more than 25 years, yet in Australia, they are not common in mathematics classes. This paper discusses the author's personal experiences at Bond University in using Microsoft Excel to assist with mathematics instruction. It goes on to present a brief overview of what other mathematics educators have said about this technology, and offers some suggestions for its incorporation into the mathematics classroom, starting at grade one.

Background

There is a software tool, on essentially every desktop or laptop computer, which is often routinely ignored for mathematical modelling and instruction. It is, of course, the modern graphical spreadsheet program. My references here to *Excel* are to Microsoft Excel 2007 or 2003 as *exemplars par excellence* of the modern electronic spreadsheet. I do this for the sake of simplicity and fully realise that some may choose to use another spreadsheet program, or none at all. One advantage of Excel is its ubiquity. Most students have already had some exposure, and the absolute basics can in most cases be assumed known. What is used in secondary schools? There seems to be a love affair with *graphics calculators* (GCs) in Australia. When superior tools such as Excel are widely available, it is difficult to understand the GC choice.

In Australia, we have significant numbers of tertiary students enrolled in subjects that, I suspect, most readers of the *Gazette* would regard as rather trivial high-school mathematics. I refer to students enrolled in BCom, BIT or similar. Here, a rather basic level of mathematics is required. While many such students have a reasonable background in mathematics, there still remains a significant proportion who do not. They have somehow escaped from secondary school with a very poor understanding of mathematical concepts, and very weak algebraic skills. At Bond University, I have often found this to be about half the class, and sometimes even more (perhaps 60%).

Teaching mathematics at Bond

Firstly, there is very little mathematics at Bond. Apart from a few units of statistics, there is only *Analytical Toolkit* (details below) and *Business Mathematics*. Secondly, Bond is private and fees are high (a typical 24-unit degree costs very close to \$72 000). This is a substantial sum, although, as public university students are now being required to shell out

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increasingly large sums for their tertiary studies, the difference has narrowed significantly in the past decade.

I feel under some obligation to assist all of my students as much as I can to reach the 'desired outcomes' for the subject I am teaching, regardless of their 'entry knowledge'. I am sure that many others also aspire to this lofty goal. However, I have essentially reached the view that, even with the best of intentions, this goal is today an impossible dream. I say this, even though my enthusiasm for teaching has not waned, and also given very small classes. The principal reasons for my negativity are that:

(1) In a large class (at Bond this is anything above about 30), I typically find several very good students, but also a (usually) much greater number of very weak students. To cover the advertised syllabus at a level with which I feel comfortable then becomes almost impossible. I usually have to omit a few sub-topics as the pace required to keep the majority of the class with me is significantly below that which I consider reasonable. Even a small rise in the level of rigour will leave half the class (roughly speaking) in the dust. Remember, they are paying around $3000 per subject, and I feel obliged to make an effort to cater for such weak students. At the same time, I try to imagine the position of the brighter students. How would a bright student of mathematics feel, having to pay such an amount if it were for a rehash of grade 11 or 12 material?

(2) The mathematical knowledge of entering students seems to decline each year. Example: this semester I am teaching a subject known as *Analytical Toolkit*. It is two-thirds discrete mathematics and one-third statistics, being the amalgam of two former units. In a lecture recently, I was about to complete an example of proof by induction, and I had the equation $2^k - 1 = 3\alpha$. 'I need to solve it for 2^k', I said to the class (only six students). 'I wish to transpose the -1. What will the right-hand side then look like?' *No-one could answer that question.* Finally, one ventured: 'the 3α will become 4α'. Note to self: I am totally wasting my time trying to teach induction. Better to omit it, and give them something that they have some chance of learning with the short 12-week Bond semester. This will be my recommendation to my Head of School at the end of semester.

Math wars

In the USA, since 1989, there has been heated debate over the introduction of a new mathematics curriculum. In brief, traditional algorithms such as those for addition and multiplication of decimal strings are de-emphasised or omitted completely. This is done in favour of allowing students to rediscover such basic mathematical truths for themselves. It is associated with Piaget's philosophy of *constructivism*. Let me very briefly state my own view. In mathematics education, I see a place for constructivism. Rediscovery of mathematical truth is exciting and rewarding for students. I do not deny that some time spent on such discovery is worthwhile, even with the limited time available. But to spend each lesson reinventing wheels is, in my view, just plain silly.

The rich body of mathematics we now possess is a legacy obtained over thousands of years. It is the result of much contemplation and experimentation by great mathematical scientists — very gifted men and women. While mathematical discovery is exciting and rewarding, shall we expect, for example, our high-school seniors, whom university lecturers will ultimately expect to have some mastery of calculus, to reinvent the work of Newton or Leibniz? Once again, this would be wonderful, and their understanding would, no doubt, be more profound, if we had the time. But we do not. My own view leans strongly toward learning of the traditional algorithms, but yes, also add some discovery. In this regard, for the purposes of both tool and catalyst for the '*Eureka!*', I use Excel. It allows for very efficient construction of models and visualisation of structures that would take students with just pen and paper, or even graphics calculators, far too long to expose. To fail to teach the fundamental algorithms of arithmetic in the early years of schooling is, in my view, a fundamental mistake. The effects are pernicious.

Amid the mountain of literature on the 'Math Wars', I have found [7] particularly lucid.

Why I use Excel in my mathematics classes

It is difficult in such a short space to convey the variety of uses to which I put Excel in my mathematics classes. My attempt at a summary appears here in the next section, while further detail may be found at the *Spreadsheets in Education* website [3]. One of the benefits of electronic journals is that live models can be downloaded by readers[1].

Because of the generally poor standard of students over the past 15 years, I have been forced to dumb-down my classes. As noted earlier, a principal component of my response to the diversity problem has been to use Excel for the illustration of mathematical concepts, and also for mathematical modelling and problem-solving. In general terms, I have found Excel to be very useful for:

- Conveying mathematical principles to students whose algebra is very deficient, and in many cases, essentially non-existent.
- Illustration of such principles to *all* students. This may include unsuspected connections between topics which had appeared to be unrelated. An example that immediately comes to mind is that of binary truth-table, sets, subsets, powersets, binomial coefficients. This example also beautifully illustrates, in Excel, the inductive step of the proof of the cardinality of 2^S [9].
- Description and demonstration of problem-solving principles/techniques and algorithms (for example, gcd).
- Providing the algebraically-able students with new material (examples include curve-fitting from first principles, simulation techniques).
- Observing patterns which may then suggest general principles or theorems. Using tables, graphs, colours and patterns to teach basics of mathematics is not the norm at Australian universities. Looking for patterns using colours and table may appear trivial. At first, I thought so. On some deeper reflection, I believe it is not. Humans are hardwired for pattern recognition. Learning and developing proficiency in mathematics makes use of this innate human facility. Some modern definitions of mathematics include the statement 'study of patterns'. As working mathematicians, we are certainly looking for patterns when we develop a conjecture which we hope to elevate to theoremhood after constructing a proof.

Topics illustrated using Excel

What are some of the topics, approaches and methods I use? Here are just a few examples.

[1] Please email me if there seems to be some implication here that a particular Excel model is online at SiE but you cannot find it. If I have it, then I shall be pleased to send it to you.

- *Sequences.* For the most basic of these, the most natural definition is the recursive one: start anywhere, then either keep adding a constant (arithmetic), or keep multiplying by a constant (geometric). In Excel, the recursive step is accomplished by a double-click. The challenge for the teacher is, of course, to relate the mathematical formalism of a recurrence relation to the readily comprehended spreadsheet notion of *fill-down.* I discussed this in [9]. Equation (1) represents a superannuation model with \$100 000 rollover (initial value), 1% interest per month, and \$500 contribution per month. It is interesting that many students have difficulty even *understanding* equation (1), let alone solving it, yet have very little trouble implementing the corresponding model in Microsoft Excel. Note: the alteration of *just one character* in the Excel model (+ to −) converts the superannuation model to a mortgage model.

$$a_n = \begin{cases} 1.01a_{n-1} + 500 & \text{if } n > 0, \\ 100,000 & \text{if } n = 0. \end{cases} \tag{1}$$

  Coupled recurrence relations (difference equations) relating to population dynamics, for example, are also easily handled [10]. These may be non-linear and/or stochastic.
- *Binary connections.* Truth table: comparison of two algorithms for constructing just the input bit-vectors (strings) is instructive. Then connections with set membership (the basic Boolean question in set theory) are examined. Each bit represents inclusion or exclusion of an element into the typical subset of a given set. This leads to easy construction of powerset for sets of modest cardinality. Students see very clearly the exponential nature of powerset, and the binomial coefficients emerge naturally [9]. The model can be extended to sampling theory: showing all possible samples of a small set, from which parameters may be computed.
- *Number theory.* Illustration of Euler's totient function $\varphi(n)$, and the number of divisors function $\tau(n)$, as well as the basic operations of modular arithmetic, including inverse, exponentiation. This culminates in a model for the RSA public key cryptosystem [11].
- *Stochastic modelling (deterministic and simulation).* Over the past 12 years I have done much consulting work on Keno. I use the more elementary parts in class to illustrate basics of probability, in particular, *expected value.* The students seem to appreciate the practical nature of the work — all done in Excel, of course.
- *Conditional formatting.* Use of *conditional formatting* to automatically highlight a cell based on its current value. The literature on the use of this feature for educational applications is scant indeed, although applications abound [2]:
  - solving $f(x) = 0$ or $f'(x) = 0$ without algebra, but by just observing change of sign (colour) [12].
  - computing modular inverse or verification of its non-existence by table lookup.
  - illustrating the solution of simultaneous linear congruences.

What have educators said about spreadsheets?

This section makes no pretence at being exhaustive or balanced — it is unashamedly biased toward positive comments about spreadsheets in education. For a much more comprehensive summary, including some negatives, see [3].

A very recent assessment is that of Haspekian [4]:

> An inventory of didactic research in this area shows the importance given by researchers to the potential of the spreadsheet for the teaching and learning of *algebra*.

Note the remark of Morishita *et al* [5]:

> ... the spreadsheet has allowed teachers to adopt a middle course, compared to the extremes of fully coding an algorithm in some programming language such as BASIC or Pascal, or using an off-the-shelf package with a canned solution. It is argued that neither of these methods is ideal for learning and the spreadsheet approach is recommended.

Neuwirth and Arganbright offer a similar viewpoint in their prologue of [6], and essentially the same points are made by Steward [8]:

> I would suggest that when both are possible, students find it easier and quicker to use a spreadsheet than write a computer program. Moreover, once written a program can often mask the mathematics that it is intended to represent, while on a spreadsheet the procedure is constantly exposed.

Sutherland and Rojano [14] investigate the potential of the spreadsheet to enable students to form a correct understanding of algebraic concepts. Sutherland [15] has much more recently used the spreadsheet environment to allow secondary school students in the UK to develop basic concepts of algebraic dependency. Since the students have trouble with the abstract nature of algebra, the spreadsheet is used to develop relationships with point-and-click [15]:

> Mouse pointing becomes a way of supporting pupils to express general relationships, which are then represented automatically in spreadsheet code. .... In this way the algebra-like spreadsheet code is learned effortlessly without explicit teaching. Pupils use the spreadsheet specific calculations to help in the construction of general rules and often verify their general rule with reference to specific numbers. In this way links between symbols and general numbers are established.

Similar comments are made by Abramovich [1]. Referring to a spreadsheet model to support an inductive proof, it is stated that the model:

> ... allows for the visualisation of an inductive proof of combinatorial identity, and it cognitively supports a transition from computing to a formal language of mathematics.

In hindsight it seems obvious, but probably one of the most profound, clear benefits of using spreadsheets is just that of *saving time*. The time gained may then be spent on investigating properties of the mathematical objects created in the spreadsheet environment.

In closing this section, I should mention the compilation of Vacher and Fratesi: a rather interesting collection of published examples of spreadsheets in the *Journal of Geoscience Education*[16].

SiE

John Baker and I created the electronic journal, *Spreadsheets in Education*, (*eJSiE*) in late 2002, with the first number appearing mid-2003. It is an open access journal: all articles are freely available from the website [3]. Content of the journal is divided into fully peer-reviewed articles and classroom articles, the latter typically accompanied by ready-to-use spreadsheet models. An annotated bibliography, up to Volume 2, #1 is given in [13].

Conclusion

Much of this article has described my response to a very undesirable situation, namely, large numbers of very poorly-prepared students in the tertiary mathematics classroom. The whole milieu is one in which mathematics is not valued, is downplayed, and treated as a necessary evil by both students and administrators. Both of these classes, directly or indirectly, now control the dollars. However, few members of either class seem to have much inkling of the true value of mathematics, as applied to a vast range of disciplines, or even what mathematics *is*. It is clear that large sections of the general public are ignorant of the broad scope of mathematics when, in the newspaper, we read '... to solve this Sudoku requires no mathematics', but fundamental logic, set theory, decision trees, *reductio ad absurdum* are all used in this game.

From the ground up, mathematics educators must become aware of the possibilities for *evangelism of mathematics*, or the public perception of our practical worth as mathematics professionals will plummet even further. I am not claiming that use of spreadsheets is a panacea, but I do suggest that spreadsheets must inevitably be part of any aggressive move by professional mathematicians as mathematics educators to reclaim ground, which, we had somewhat smugly assumed would never be ceded. It has slipped away. We must restore rigour, but shall inevitably do so in an environment where many of the teachers and students are themselves products of a flawed system. Rightly used, spreadsheets can help us.

We are in the 21st century, and like it or not, students (or their parents) paying the dollars expect Internet, video/DVD images and much more digital magic. They are wondering: how can boring old maths still be of any relevance? It is up to us to persuade them that it is. We sorely need *mathematical evangelists*. Of course, *Gazette* readers know that mathematics is more relevant than ever, being the very foundation of any quantitative science, including the basis of all modern IT wizardry, computer science! How could anyone seriously deny the value of mathematics? Well, the rot set in around 20 years ago, and we are now in dire straits, brought on by years of compounded negative effects of flawed federal and state funding systems, and our own complacency. We sat by and blithely assumed that no rational person could seriously deny the value of mathematics. We need positive moves, where benefits are made clear. I submit that spreadsheets can help enormously, as they offer a friendly vehicle for support of mathematics learning in the very earliest years of schooling.

I have put forth a very brief case for the use of electronic spreadsheets at all levels of mathematics education, but especially in the early years, right from grade 1. There is plenty of literature to support my case. My own view is that the case is clear: there is no longer any need to justify the value of spreadsheets for mathematics education — it should be obvious to anyone who cares to objectively examine the evidence. The next step is to convince the practitioners, decision makers and those who hold the purse-strings. We do not need to convince the students — they are already on board. Indeed, many of them are probably wondering why we, so often, fail to take advantage of such a useful and ubiquitous tool in the mathematics classroom.

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Steve Sugden is Associate Professor (Mathematics and Computation) at Bond University. He obtained his PhD in Operations Research (nonlinear integer programming) from Bond in 1992, working under the supervision of Professor Bruce Murtagh, creator of the AESOP and MINOS optimization packages. His scientific career has been rather diverse, with significant periods in industry, working in areas as diverse as solar energy, technical engineering software for various Queensland electricity authorities, and cryptographic software engineering. His present research interests at Bond centre on the use of technology in mathematics education, especially modern spreadsheet programs such as Microsoft's Excel. In 2002, he established the electronic journal *Spreadsheets in Education*, hosted at Bond. This journal publishes fully peer-reviewed articles plus classroom resources for teachers. In recent years, he has also been active in consulting work and has developed several mathematical models relating to aspects of Keno.

Puzzle corner 2

Norman Do*

Welcome to the Australian Mathematical Society *Gazette*'s Puzzle Corner. Each issue will include a handful of entertaining puzzles for adventurous readers to try. The puzzles cover a range of difficulties, come from a variety of topics, and require a minimum of mathematical prerequisites to be solved. And should you happen to be ingenious enough to solve one of them, then the first thing you should do is send your solution to us.

In each Puzzle Corner, the reader with the best submission will receive a book voucher to the value of $50, not to mention fame, glory and unlimited bragging rights! Entries are judged on the following criteria, in decreasing order of importance: accuracy, elegance, difficulty, and the number of correct solutions submitted. Please note that the judges decision — that is, my decision — is absolutely final. Please e-mail solutions to N.Do@ms.unimelb.edu.au or send paper entries to: Gazette of the AustMS, Birgit Loch, Department of Mathematics and Computing, University of Southern Queensland, Toowoomba, Qld 4350, Australia.

The deadline for submission of solutions for Puzzle Corner 2 is 1 July 2007. The solutions to Puzzle Corner 2 will appear in Puzzle Corner 4 in the September 2007 issue of the *Gazette*.

Coffee and doughnuts

In a certain mathematics class, if each boy purchases a coffee and each girl purchases a doughnut (all items costing an integer number of dollars), the class would spend a total of one dollar more than if each boy purchased a doughnut and each girl purchased a coffee. If there are more girls than boys in the class, what can one determine about the number of girls and the number of boys?

Solitaire

On an infinite chessboard, a game is played as follows. At the start, n^2 pieces are arranged on the chessboard in an $n \times n$ block of adjoining squares, one piece in each square. A move in the game is a jump in a horizontal or vertical direction over an adjacent occupied square to an unoccupied square immediately beyond. The piece which has been jumped over is then removed. Find those values of n for which the game can end with only one piece remaining on the board.

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Glass half full

You have a transparent glass which is in the shape of a right cylinder and it appears to be approximately half full of water. How can you accurately determine whether the glass is half full, less than half full, or more than half full, using as little equipment as possible?

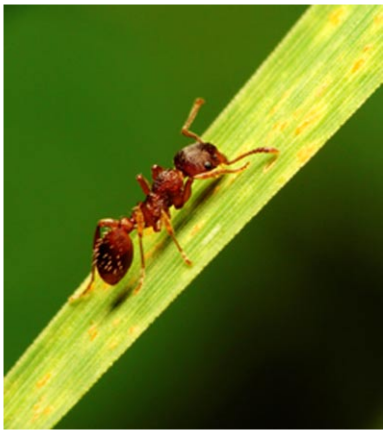
Secret salaries

Three mathematicians would like to know the average value of their salaries. How can they all determine the average without disclosing the value of their own salaries to each other?

Ambulatory ants

(1) A number of ants are distributed around a narrow circular path. At one particular instant, each of the ants chooses a direction and begins to walk along the path. The ants all walk at the same constant speed and, when two ants meet, they both instantaneously change directions and continue walking at the same speed.

Prove that at some later moment, every ant will be in its starting location.



(2) A number of ants are distributed along a thin stick one metre in length. At one particular instant, each of the ants chooses a direction and begins to walk along the stick. The ants all walk at the same constant speed and, when two ants meets, they both instantaneously change directions and continue walking at the same speed. Furthermore, when an ant meets an end of the stick, it instantaneously turns around and continues walking at the same speed.

(a) Prove that after two minutes, every ant will be in its starting location.

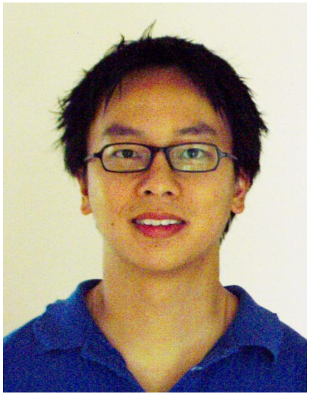
(b) Under what conditions is every ant in its starting location after one minute?

(3) Three ants find themselves on the hour, minute and second hands of an analog clock at noon. After that point in time, whenever one hand of the clock overtakes another, the two corresponding ants instantaneously swap positions. Twelve hours later, each of the ants has travelled a whole number of revolutions around the clock. Which ant has travelled the most number of times around?

An unusual identity

For a fixed value of n, choose any subset of n integers from the set $\{1, 2, 3, \ldots, 2n\}$. Now arrange them in increasing order to obtain the sequence $a_1 < a_2 < \cdots < a_n$ and arrange the remaining numbers in decreasing order to obtain the sequence $b_1 > b_2 > \cdots > b_n$. Determine all possible values of the expression

$$|a_1 - b_1| + |a_2 - b_2| + \cdots + |a_n - b_n|.$$



Norman is a PhD student in the Department of Mathematics and Statistics at The University of Melbourne. His research is in geometry and topology, with a particular emphasis on the study of moduli spaces of algebraic curves.

The style files

Appearance affects communication; but not necessarily as you like

Tony Roberts[*]

Many people are trained to produce documents that look appealing. For example, many favour the supposedly clean appearance of a sans serif font such as the mainstream Arial. However, research by Australian Colin Wheildon [6] showed that a miserable 12% of readers comprehended a text in a sans serif font; in comparison, 67% of readers comprehended the same text when it was typeset in a serif font (as is the bulk of this document). Not only does a supposedly pretty or clean appearance not equate to effective communication, it is far different.

Decide now whether you are more interested in your own subjective opinion of the look of your document, or whether you are more interested in how to format your document so that *others can most easily comprehend your writing.*

If the former, then stop reading now. I write here only for those who wish to learn to effectively communicate.

Short solution

Wheildon's [6] research shows we should use serif fonts such as Computer Modern or Times. Is there a simple solution to implement for your documents a style that is effective in communication? Silly question: of course there is. The short answer is to *use LaTeX and accept all the defaults of LaTeX.*

Knuth and Lamport consulted many professional printers to find out what they did and why. Knuth and Lamport then encoded into LaTeX the wisdom of centuries of experience in printing. I know many have difficulty accepting this: nonetheless, accept that LaTeX knows best.

Slightly better solution

Priestly [4] comments that page layout, especially for instructional documents, should best be in two columns with the right column for the main text and the left column for headings, major points and prompts. Fortunately, Hubert Partl and Axel Kielhorn implement such a two-column layout for us in their LaTeX classes refart.cls and refrep.cls for articles and reports respectively[1].

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[1]Download refman.dtx from any ctan site and install.

Apart from font and a left column for headings, what other aspects help readers comprehend our documents? I summarise here some of the aspects reported by Priestly [4] and based upon research into effective communication. I emphasise again, you should prefer such a style not because it might or might not look pretty, but because research demonstrates the style is most effective for comprehension.

Line width

On average, each line should have 10 to 12 words, or equivalently, be roughly 60 characters wide. Human eyes do not scan well wider lines[2]. But we want to save trees by having as much text per sheet of paper as possible. I offer two solutions: either typeset in two columns utilising the whole page; or typeset as a document on A5 paper[3] and print two A5 pages per sheet of A4 paper.

Text colour

As Priestly [4] puts it: use any colour so long as it is black. For example, although eight out of ten people consider blue text more attractive than black text, give them a couple of pages to read and their comprehension tells a different story: in one test 70% of readers of black text showed good comprehension, whereas barely 10% of readers showed good comprehension of the same text when it was coloured blue. Colour attracts the eye and can be good for headings, but colour is woeful for comprehending text.

Emphasise discreetly

Modern computer publication allows us almost infinite variety in style. Many writers adopt variety with enthusiasm, but at the unseen cost of confusing their readers. Here are some rules of thumb.

- Use bold only for navigation: bold text is much less readable. In one test, 70% of readers comprehended a text in ordinary font, but only 30% of readers comprehended the text in bold. Bold font attracts the eye and thus in headings and definitions usefully helps a reader to navigate around a document. Bold is not useful for comprehending sentences.
- Never use all capitals: we recognise words partly by the shape of their outline, and all capitals destroys that shape; use lower case.
- Similarly avoid underlining, reverse type, and outline type.
- Italic font also degrades the word image and thus interferes with reading. Use italics when you emphasise, but use it sparingly.
- Placement is your most effective tool for emphasis. Ensure that: your most important sentences are at the start or end of each paragraph; your most important paragraphs are at the start and end of each section.

Summary

Learn to love the default style of LaTeX: it is close to being the best that research shows is effective for written communication.

[2]This is why LaTeX in 10-point font has more characters per line than LaTeX in the larger 12-point font.
[3]Use the a5paper option in the geometry package.

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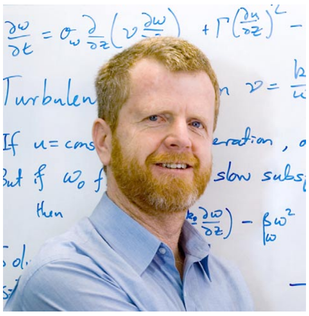


Tony Roberts is the world leader in using and further developing a branch of modern dynamical systems theory, in conjunction with new computer algebra algorithms, to derive mathematical models of complex systems. After a couple of decades of writing poorly, both Higham's sensible book on writing and Roberts' role as electronic editor for the Australian Mathematical Society impelled him to not only incorporate writing skills into both undergraduate and postgraduate programs, but to encourage colleagues to use simple rules to improve their own writing.

Communications

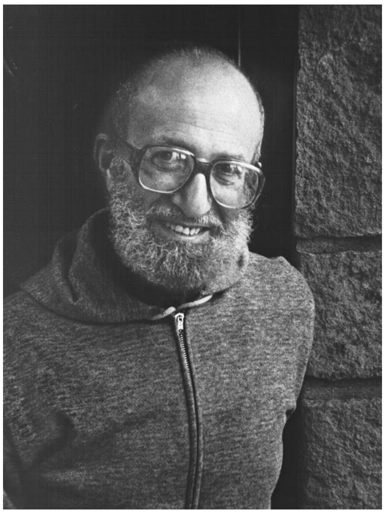
Paul Halmos and the Macquarie Annual Meeting

On 25–29 September 2006, at Macquarie University, the Society held its 50th Annual Meeting. Several days later, on 2 October, Paul R. Halmos, one of the most striking personalities of 20th century mathematics, passed away in California, USA. Halmos had been an invited speaker at a previous Annual Meeting at Macquarie: the Society's 33rd, in the week of 3–7 July 1989. Three present-day Macquarie mathematicians have related their reminiscences of Halmos in connection with that earlier conference.

John Corbett: When Halmos came to Australia for the Macquarie conference in 1989, he also gave a talk in the University of NSW Mathematics Department. I was in the audience, sitting near the back of the room, and before he spoke, Halmos spent a few moments looking at the faces before him. He quietly walked up to me, shook my hand and, remembering my name, asked me how I was. I was flattered but I don't know whether anyone else noticed. It was very kind of him.

Ross Street: The advertised topic of Halmos' talk in the 1989 conference was, 'Fifty years of linear algebra: A personal reminiscence'. I recall that he didn't wish to provide an abstract for his talk; I think this was one of his general principles, and that he felt an abstract would reduce the impact of a good talk.

Halmos came to Australia on at least one other occasion. His passion for taking photos of mathematicians, and for remembering names, became very obvious during these visits. Especially in view of John Corbett's comment above, I feel quite distinguished because Halmos could not remember me from previous occasions when we'd met. Yet Halmos had his own lack-of-recognition story, which I think went like this. He was in a lift one day, at a US university, when his colleague, André Weil, mistaking him for the lift operator, turned to Halmos to tell him the floor they wanted to go to!



Paul Halmos

Mike Johnson: I particularly remember Halmos at the 1989 Macquarie conference, because his presence was very nearly responsible for what might have been my greatest claim to fame in mathematics. Early one morning, hurrying into the university to set up for the first session, I was driving quite fast down the entirely clear curbside lane of Herring Road, when, from between cars queued in the centre lane (waiting to turn right to Macquarie Centre), hobbled Halmos. He seemed to move very quickly, on his walking stick, from between the stationary cars; but to walk painfully slowly once he was in my lane. With considerable effort I missed him, very narrowly. He was a lovely bloke, and I'm very glad he lasted another 17 years, and that I avoided being known as the mathematician who killed Halmos!

National Strategic Review of Mathematical Sciences Research in Australia

Hyam Rubinstein* and Jan Thomas**

Concern over 'failing health' of Australia's mathematical sciences

In an unprecedented move, the international mathematics and statistics community has rallied in support of their Australian colleagues and signed an open letter to Prime Minister John Howard (see opposite), urging him to prevent the imminent collapse of our national mathematical sciences capability. More than 110 of the world's leading mathematicians and statisticians and almost 400 Australians have signed the open letter, calling on the Prime Minister to address the 'perilous path' of Australian mathematics and statistics as a matter of urgency.

The open letter was circulated three months after the release of the National Strategic Review of Mathematical Sciences Research in Australia, in which business leaders, top government agencies, universities and mathematicians and statisticians warned that the system was near collapse. Although the issue has received widespread media coverage since the release of the review, the Australian Government has so far failed to take any action to implement the key recommendations.

Worryingly, since the review was released, matters have further deteriorated, with the already seriously depleted base of mathematical sciences in Australian universities being further eroded, with a number of universities currently reducing staff through voluntary or forced redundancies.

Many of the world's mathematical sciences leaders including Sir John Ball (President of the International Mathematical Union), Professor Terence Tao (Fields medallist), and Sir Michael Atiyah (Fields medallist) signed the letter.

Australians who signed the letter included academic mathematicians and statisticians, teachers, engineers and geophysicists, many from medical research and others from diverse fields. They included Professor Suzanne Cory (Director, Walter and Eliza Hall Institute of Medical Research) and Professor Ian Sloan (President of the International Council for Industrial and Applied Mathematics).

On behalf of the Australian mathematical sciences community and the Review Working Party, we thank them all.

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An Open Letter to the Prime Minister of Australia

Australia has had an international reputation for excellence in research and teaching in mathematical sciences and has produced many fine mathematicians and statisticians. This reputation was further enhanced when Professor Terry Tao was awarded the Fields medal in 2006.

The findings of the recent National Strategic Review of Mathematical Sciences Research in Australia, completed in December 2006, are therefore deeply disturbing. Australia cannot continue to make this kind of contribution from its shrinking research base and narrow concentration of research fields. The Review found that the shortage of mathematical modellers and statisticians is so severe that it inhibits work of business and industry, such as mining and biotechnology companies and financial institutions, and government agencies including the Australian Antarctic Division, the Australian Bureau of Statistics and CSIRO. The shortage brings pause to foreign companies' plans for investment in Australia.

The collapse of Australia's mathematical sciences departments also prevents them from educating the mathematics teachers that are so desperately needed by the nation's schools.

The three distinguished international reviewers, Professor Jean-Pierre Bourguignon, Dr Brenda Dietrich and Professor Iain Johnstone, found 'the nation's tradition and capability to be on a truly perilous path'.

Even more disturbing is that, in the short time since the Review, the already seriously depleted base of mathematical sciences in Australian universities is being further eroded with a number of universities currently reducing staff through voluntary or forced redundancies.

We have noted your commitment, quoted in the Review, that 'in this ever more competitive global economy, Australia's science, engineering and technology skills need to match the best in the world'. Science, engineering and technology skills depend on mathematical sciences. We, the undersigned, urge the Australian government to take urgent action and immediately address the priority areas identified in the Review.

Assessing research in the mathematical sciences

Alan L. Carey*, Michael G. Cowling** and Peter G. Taylor***

In the context of the forthcoming Research Quality Framework (RQF), we discuss the assessment of the quality of research in mathematics and statistics. The purpose of this document is two-fold: to act as a resource for RQF panel members and to help group leaders prepare context statements, as the RQF will require them to do. However, the discussion of assessment that we propose is equally valid for the purposes of hiring and of promotion.

Introduction

The recommended procedures for the Australian Research Quality Framework (RQF) were released in October 2006 [5]. Under these procedures, research in Australia will be assessed by a number of assessment panels. While the most relevant panel to the mathematical sciences is 'Panel 4: Mathematical and Information Sciences and Technology', a number of other panels are also relevant, particularly for statisticians and applied mathematicians.

This document aims to indicate the many ways in which mathematical sciences research can be and is evaluated internationally. Essentially, it is a statement about the research culture of the mathematics and statistics community. Our hope is that this will be a guide to assessors on RQF panels on how to judge the quality of research in the mathematical sciences. We also intend this document to be of assistance to researchers in drawing up context statements, as detailed in [5, Section 4.1.5].

Primarily, the mathematical sciences are described by Research Field Courses and Disciplines Classification (RFCD) Codes 2301 (mathematics), 2302 (statistics), 2399 (other math sciences) and 2804 (computation theory and mathematics). However, these codes do not cover all research output in the mathematical sciences, which also appears under the RFCD codes associated to many other disciplines, including the various branches of engineering and theoretical physics. Further, mathematical, and particularly statistical, research is also published in journals related to biological, medical and agricultural science, economics and in many parts of social science.

There are vastly different research cultures in the mathematical sciences, and even within various subdisciplines of mathematics and statistics. To give some extreme examples, Andrew Wiles published an average of one paper a year over 13 years before he proved Fermat's Last Theorem, and yet his achievement is arguably the most significant mathematical result of the late twentieth century. The great logician Gödel's research output consisted of

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half a dozen papers. On the other hand, Paul Erdős published over 1500 papers, the great majority in collaboration with colleagues all over the world, and the polymath Leonhard Euler wrote close to one thousand papers. However, these extremes are not representative, and in most areas of the mathematical sciences, a publication rate of one to five papers per year is considered 'normal'. There are significant variations between the norms of the many subfields.

Because of this variability, most mathematicians and statisticians argue that it is dangerous to use bibliometric data without first attempting to understand the culture of the relevant subdiscipline. Indeed, as argued comprehensively in [4], the use of crude measures of productivity based on standards of related but distinct disciplines is likely to reduce quality in the long term. Most mathematicians and statisticians agree that it is important to use a much wider range of indicators than just publications.

Publications

Most mathematicians and statisticians support the principle espoused in [5] that quality is best assessed via disinterested peer assessment of research outputs. In particular, there is widespread support for the principle that high quality publications are the primary indicators of research strength, and that assessors should read some of the papers or books that have been produced, not just rely on scientometric measures such as ISI journal impact factors.

Mathematicians and statisticians produce several types of outputs. The most common are papers in refereed journals with an international editorial process and international circulation. Articles are also published in conference proceedings, frequently as a result of mathematicians and statisticians working close to a field of application and publishing according to the culture of that field. For example, a mathematician who collaborates with engineers or computer scientists might publish in conference proceedings in these areas, which are often amongst the most prestigious outlets available.

In contrast, many mathematics conferences do not publish proceedings at all. Indeed, a common characteristic of the most prestigious conferences in mathematics and statistics is that speakers are expected to present their work and then publish it in fully-refereed journals. Scholarly books are only rarely produced.

The ordering of the authors is frequently judged to be important by those compiling quantitative measures of research output. In mathematics and statistics, it is very common, but not universal, to order authors alphabetically (this may disadvantage authors whose surnames begin with later letters in the alphabet [1], but there does not appear to have been a complete analysis of this). Other systems that are used by some people include ordering authors by the level of contribution, going from highest to lowest, and putting graduate students first. The results reported in a joint paper are frequently the product of 'brainstorming' sessions attended by all authors, and in this context, it is often argued that the result would not have been produced if there had not been a contribution from all authors, and that attempting to apportion different proportions of the idea to each of the authors is a fruitless exercise.

It is virtually unknown for mathematicians to be listed as authors on papers to which they have not made substantial intellectual contributions; this contrasts with many laboratory disciplines, where researchers' names are often included in the list of authors by virtue of their position in a laboratory. As a result, mathematicians tend to be listed as authors of fewer papers than their colleagues in the experimental sciences. Again in contrast to many other disciplines, papers in the mathematical sciences do not even attempt to compile an exhaustive bibliography of all relevant papers. Rather, a paper will be cited because a result contained therein is needed. Particularly coupled with the fact that in many subdisciplines of mathematics publication is infrequent, this means that numbers of citations of a paper in the mathematical sciences is generally lower than that of a paper in many other sciences. This in turn leads to scientometric indices such as impact factors of mathematical sciences journals being lower than those of other scientific disciplines.

In the mathematical sciences, there can be a considerable time-lag, typically between one-and-a-half and two years, between manuscript submission and subsequent publication. This should be kept in mind, especially when the performance of an early career researcher is assessed. An important consequence of this lag is that the ISI journal impact factor is not a robust measure of a journal's standing, since it only takes into account the number of citations in the two years following publication. In fact some of the most prestigious mathematics journals have low impact according to this measure, and rankings by impact can vary widely from year to year: for instance, in the 'Mathematics' ranking by impact factor of some 120 journals, the *Publications Mathématiques de l'Institute des Hautes Etudes Scientifiques* was 100th in 1989 and first in 1990.

There is, however, a generally accepted crude ranking of mathematics and statistics journals within discipline areas in terms of their quality, which is quite well correlated with impact when this is measured over decades rather than years. Expert opinion can advise on this. For more on publication patterns in mathematics and the evaluation of journals, see [2], [3].

Grant Funding

The major use of grant money in mathematics and statistics is to fund employment of postdoctoral fellows and other staff. The existence of such funding is essential for the development of the next generation of practitioners in the disciplines, so it is very important that mathematicians engage in the grant application process.

It is possible for some mathematicians to pursue their research without engaging vigorously in the grant process. Despite this, we would argue strongly that grant success should be applied as a measure of research productivity. It is a good indicator of the esteem that researchers are held in by their peers. Moreover, a person who has a long and consistent record of grant success is very likely to be a research leader in the sense that they will have supervised and mentored a number of postdoctoral fellows. The most common granting scheme accessed by Australian mathematicians is the ARC Discovery Grant Scheme. Applied mathematicians and statisticians are also able to access the ARC Linkage Grant Scheme. International funding is also becoming a prestigious source of support for research in the mathematical sciences.

One consequence of the fact that mathematicians and statisticians generally do not need expensive experimental equipment is that they generally apply for less funding per application than other scientific and technological disciplines. It is therefore inappropriate to judge grant success by looking at the total funding earned, especially if this is compared with researchers from disciplines that do use expensive equipment. A better measure is the number of successful applications, or the rate at which a researcher achieves success.

Postgraduate research student training

An important role of academics in all fields is their contribution to the development of higher degree students. A good measure of the effectiveness of a higher degree program is the proportion of students who graduate in a timely fashion and go on to employment (including further study) in the area of their study and research.

As with other measures discussed above, it is important to assess research student supervision in a manner that is appropriate to the discipline. Typically, a supervisor–student relationship in mathematics or statistics resembles those that occur in the humanities more than those in the laboratory sciences. Supervision is frequently one-to-one between student and supervisor. Joint supervision is becoming less unusual but, even then, all supervisors have to keep on top of the detail of the intellectual content.

It is rare for students to work on a research problem that is a small part of a large project that a team, including other students, is working on. On the contrary, a supervisor may simultaneously look after students who are working on several very different projects, which require different sorts of intellectual input.

These factors (and others) mean that the rate of production of PhD graduates in the mathematical sciences is lower than in many other scientific disciplines. Overall, there have been less than 1500 PhD graduates in the mathematical sciences in the entire history of the Australian university system and the number of people who have supervised more than ten students is quite small. The contribution to student supervision of such people ought to be regarded very highly. Mid-career researchers who have supervised between five and ten successful PhD students ought to be well-regarded for their contribution to supervision.

Further indicators of esteem

There are a number of other measures that are good indicators of research standing in the mathematical sciences community. These include:

- Invited conference talks at highly selective and prestigious venues. These include large conferences, such as the International Congress of Mathematicians, the International Congress of Industrial and Applied Mathematics, and the International Congress on Mathematical Physics, as well as smaller focussed events such as those held at the Mathematisches Forschungsinstitut at Oberwolfach in Germany.
- Invited fully- or partly-funded visits to leading research centres and institutes such as the Newton, Erwin Schrödinger and Mittag-Leffler Institutes in Europe or the Mathematical Sciences Research Institute, the Institute for Mathematics and its Applications and the Fields Institute in North America.
- Prizes, fellowships and awards, particularly those won in international competition.
- The quality and extent of researchers' collaborations is often taken as a good measure of their standing in the mathematical sciences community. These frequently take the form of links with international leaders of the discipline or subdiscipline. For applied research, substantial industrial collaborations provide an analogous indicator.
- Membership of editorial boards of international journals.
- Membership of the organising committee or advisory board of prestigious international conferences.
- Scholarly activity such as reviewing and refereeing.
- Assessing research theses and research grant applications, especially if in another country.

- The extent to which researchers' work influences the work of other researchers.
- Production of (documentably) high-quality and widely-used software packages.

A critique of citation analysis

In this section we shall make some points about citations in the mathematical sciences.

First, to make bibliometric assessments, citation data from many sources is needed. In particular, ISI data misses articles that appear on the web where, in many fields of research, the most intense citation activity occurs prior to actual publication. Other sources, such as Google Scholar, www.arxiv.org and MathSciNet, all provide different information.

It is noteworthy that the papers for which Tao and Perelman won Fields Medals (the mathematical equivalent of the Nobel Prize) in 2006 do not show up in ISI citation data, as they were still in electronic form at the time of the award. The paper which led to the Fields Medal for Simon Donaldson had about 80 ISI citations at the time of the award, while many Nobel Prize winning works have thousands of citations.

Second, in interpreting citation data of a research output, it is essential to understand the citation culture of the subdiscipline that provides the audience for the research. There are wide variations of culture within a single four-digit RFCD code, and agglomerating data from groups with different cultures will mean that differences of quality within a subdiscipline will be swamped by differences in culture between subdisciplines.

Third, to assess the importance of a research article, the way it influences research will ideally be considered over its full life-time. This life-time varies within the various subdisciplines of the mathematical sciences, but it is common for the 'citation half-life' of an article to be over 10 years. It is certainly hard to assess correctly the long-term value of a mathematical contribution until many years after its appearance, and the RQF proposal to judge research output from a comparatively short time window is problematic for the mathematical sciences.

Fourth, the ISI classifies the mathematical sciences into 'fundamental' and 'applied' based on the labels that journals apply to themselves. This may be misleading; for example, an assessor might need to consider the citation record of papers that are genuinely 'cross-disciplinary' where the impact within the mathematical sciences may be small but the impact in another discipline may be large. Arrow's theorem in economics, a Nobel prize winning piece of research, that has not led to substantial new mathematics, is an example of such a phenomenon.

Finally, it is essential to distinguish original research papers from surveys or review articles. This does not occur with naive bibliometric analysis.

There is concern about the use of bibliometric analysis in many disciplines other than the mathematical sciences, and even suggestions that such indices are already being manipulated. See for instance [6], [7].

Acknowledgements

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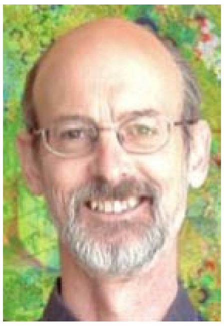
Alan Carey


Michael Cowling


Peter Taylor

Alan Carey studied at the Universities of Sydney and Oxford, then took up research fellowships at the University of Adelaide and the ANU. He held a continuing position at the University of Adelaide, with brief appointments to Flinders University and to ANU, from the mid-80s until 2002. Since then he has been the Dean of the Mathematical Sciences Institute at ANU. He has been a Clay Mathematics Institute Scholar on three occasions, in 2000 and again in 2001 at Harvard, and in 2006 at the Erwin Schrödinger Institute in Vienna. He was President of the AustMS from 2000 to 2002.

Michael Cowling studied at ANU and Flinders University, then spent a decade in Canada, Italy and the USA before returning to Australia as Professor of Mathematics at UNSW in 1983. Career highlights since then include the Medal of the Australian Mathematical Society, a 10-year stint as an ARC Senior Research Fellow, and two years as President of the Australian Mathematical Society. He is currently Head of the School of Mathematics and Statistics at UNSW, and Editor of the Journal of the Australian Mathematical Society.

Peter Taylor studied at the University of Adelaide in the 1980s, and then spent time at the Universities of Western Australia and Adelaide before moving to Melbourne at the beginning of 2002. In January 2003, he became the first Professor of Operations Research at the University of Melbourne. He is currently a chief investigator of the ARC Centre of Excellence in Mathematical and Statistical Modelling of Complex Systems, President of ANZIAM, Head of the Department of Mathematics and Statistics at the University of Melbourne, and the Editor-in-Chief of 'Stochastic Models'.

An Olympiad problem appeal

Hans Lausch*

Call for problem donation

Once every three years the Senior Problems Committee of the Australian Mathematical Olympiad Committee (AMOC) turns to our mathematical community at large with an appeal for problem donations that can be used in national, regional and international senior-secondary-school mathematics competitions. The latest appeal [1] provided examples of competition problems that had been set for various contests in Australia and in the Asia-Pacific region between 2001 and 2003. The present article is to repeat this exercise with problems from competitions held between 2004 and early 2007. Problems chosen for these competitions are from 'pre-calculus' areas such as number theory, geometry (with a strong preference for 'Euclidean' geometry), algebra, discrete mathematics, inequalities and functional equations. Here are some examples of problems used in contests held in 2004–2007.

(1) The AMOC Senior Contest is held in August of each year. About 100 students, most of them in Year 11, are given five problems and four hours to solve them. The following problem, Question 1 of the 2004 contest, has received the difficulty rating 'easy-to-medium':

> Consider eight points in a plane consisting of the four vertices of a square and the four midpoints of its edges. Each point is randomly coloured red, green, or blue with equal probability.
>
> Show that there is a more than a 50% chance of obtaining a triangle whose vertices are three of these points coloured red.

(2) Question 2 of the 2004 AMOC Senior Contest turned out to be 'hard':

> Let $a_1, a_2, \ldots, a_{2004}$ be any non-negative real numbers such that $a_1 \geq a_2 \geq \cdots \geq a_{2004}$ and $a_1 + a_2 + \cdots + a_{2004} \leq 1$.
>
> Prove that $a_1^2 + 3a_2^2 + 5a_3^2 + 7a_4^2 + \cdots + 4007a_{2004}^2 \leq 1$.

(3) The Australian Mathematical Olympiad (AMO) is a two-day event in February with about 100 participants. On either day, students are given a four-hour paper containing four problems. Students found the following problem, Question 5 of the 2006 AMO, quite easy:

> Let $ABCD$ be a square, and let E be a point on its diagonal BD. Suppose that O_1 is the centre of the circle passing through ABE and O_2 is the centre of the circle passing through ADE.
>
> Show that AO_1EO_2 is a square.

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(4) Here is a 'medium-to-hard' geometry problem from the 2007 AMO (Question 7):

> Let ABC be an acute angled triangle. For each point M on the segment AC, let S_1 be the circle through A, B and M, and let S_2 be the circle through M, B and C. Let D_1 be the disk bounded by S_1, and let D_2 be the disk bounded by S_2.
>
> Show that the area of the intersection of D_1 and D_2 is smallest when BM is perpendicular to AC.

(5) Note that the difficulty ratings of the above problems were arrived at from hindsight. In reality, problems committees often find it rather hard to pin down the difficulty of a problem when setting it. This phenomenon is well known not only on national competitions but also in regional and international competitions. Here is an example.

The Asian Pacific Mathematics Olympiad (APMO) takes place in March. The contest is a four-hour event with five problems to be solved. About 20 countries, most of them from the Pacific Rim, take part in the APMO. Usually, 25 to 30 Australian students are invited to participate in this competition. At the 2006 APMO, Question 5 was considered by the APMO Problems Committee, which includes mathematicians from three different countries, as the most difficult one on the paper. A significantly large percentage of participants, however, contradicted the committee by providing perfect or nearly perfect solutions. Here is the problem, which had been submitted by the Australian problems committee:

> In a circus there are n clowns who dress and paint themselves up using a selection of 12 distinct colours. Each clown is required to use at least five different colours. One day, the ringmaster of the circus orders that no two clowns have the same set of colours and no more than 20 clowns may use any one particular colour.
>
> Find the largest number n of clowns that the circus could employ so as to make the ringmaster's order possible.

Problems of recent International Mathematical Olympiads continue to be reported regularly in the *Australian Mathematical Society Gazette*. The complete set of AMO problems and solutions covering the period 1979–1995 can be found in [2], whereas the problems and solutions of all APMOs between 1989 and 2000 have appeared in [3], while the problems, including solutions and statistics, of each year's AMOC Senior Contest, the AMO, the APMO, the International Mathematical Olympiad and some intermediate-secondary school mathematics competitions are available in the AMOC's year books [4].

Problem donations will be gratefully received by me as Chair of the AMOC Senior Problems Committee and credit to the donor of successful problems will be given in [4].

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The African Institute for Mathematical Sciences

Kevin Burrage*

I am not sure how many of the readers of the *Gazette* are aware of the African Institute for Mathematical Sciences (AIMS), based at Muizenberg in South Africa. Muizenberg is a small beach community about half an hour's drive from Cape Town CBD, situated on the eastern coast of the Cape of Good Hope. AIMS has gorgeous views down the Coast of the Cape Hope Ranges (an extension of Table Mountain) and across False Bay. AIMS was established in 2003 under the directorship of Fritz Hahne, formerly Dean of Science at the University of Stellenbosch. AIMS is housed in an old hotel that was built in the early 20th century, and which has been substantially renovated.

Students and tutors, 2006–2007

Its mission is to promote Mathematics and Science in Africa and to provide a focal point for Mathematics university training in Africa. It offers scholarships for up to 50 students to come and study for a period of nine months. Of the 50 students, about 15 positions are reserved for females. In the 2006/2007 intake there were over 250 applicants.

The students are housed and fed and their return travel from their home town is fully funded. Lecturers also stay at AIMS and share their meals with the students, so that a rapport quickly develops. The students are away from their families and friends for nine months and are absolutely committed to the discipline of Mathematics. When they first arrive, some of them have little ability in English but since all tuition is in English they quickly learn. Some find the transitions difficult but they all support one another and at the end of their time their English skills are very good. The students do a series of subjects that last for about three weeks each, consisting of 30 contact hours, as well as a thesis/project. Each course has a number of assignments associated with it and these get evaluated. AIMS has seven or eight teaching assistants who help with the tutorials, marking, advice, and who are a vital component of AIMS.

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It is a pleasure to teach these students. They seem to prefer the blackboard style of presentation and copy all the notes down. In the classes they are very responsive. If they do not understand something they will ask questions, if they do understand they will murmur assent. This means that the lecturer ends up presenting in a very interactive way and ends up responding to the students rather than vice-versa, as in Australia.

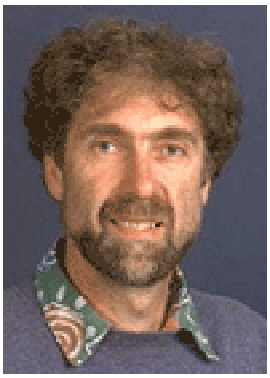
The students' backgrounds are quite varied. Two countries that seemed to have very strong students (at least while I was there) are Nigeria and Madagascar. One of the students, a young female student, is from the Darfur, so you can imagine the challenges that must be met in order to make a career there in Mathematics.

Kevin Burrage (on left) with students at AIMS in March 2007

The way it works if you would like to teach at AIMS is that academics go to the AIMS website (http://www.aims.ac.za) and fill in a form saying what topic they would like to present and when. The courses start in October and go through to the end of March. AIMS is willing to cover travel costs but always appreciates if lecturers can pay their own way. I was the first Australian to teach at AIMS — my course being on Computational Biology.

I can recommend the experience whole heartedly. It is very rewarding and gives a greater appreciation of how students in other countries, not as wealthy as Australia, struggle to make a career in this profession. Their dedication to their chosen career is marvellous to see. Most of these students want to carry on to do PhDs and many choose Europe as the fees are low and many scholarships are available. Many of them had little knowledge of Australia and were very keen to study here until I told them the fee costs!

AIMS is looking to help establish mathematical institutes in other African countries but the struggle is, as always, in the financing. I also hope that they can develop strong relationships with Australia. I wish AIMS all the luck in the world and if they do succeed in setting up other institutes in Africa I will be volunteering again.



Kevin Burrage is Professor of Computational Mathematics at the University of Queensland and Federation Fellow of the ARC. For the last five years his research has been in Computational and Systems Biology and, in particular, on the roles of noise in Cell Biology.

Obituary

Max Kelly, FAA
5 June 1930 – 26 January 2007

Gregory Maxwell Kelly was solely responsible for introducing category theory into Australia at a time when the subject was in its infancy. The Eilenberg–Kelly monograph *Closed Categories* of 1966 set the stage for two more generations of Australian category theorists. This research stream reached maturity with Max's book *Basic Concepts of Enriched Category Theory* (CUP 1982), and now finds application in many areas of mathematics, theoretical physics, computer architecture, software design, and information management.

As a student of the Marist Brothers at Bondi, Max topped the NSW Leaving Certificate overall. He went on to win, in 1951, the University Medal for Mathematics at the University of Sydney and to gain the James King of Irrawang Travelling Scholarship to study at Cambridge. There he obtained a BA with First Class Honours and two Wright's Prizes in 1953, a Rayleigh Prize in 1955, and PhD in 1957; the doctorate was in algebraic topology under the supervision of Shaun Wylie. Max returned to the University of Sydney in early 1957 as a Lecturer in Pure Mathematics and was promoted to Senior Lecturer in 1961 and to Reader in 1965.

My first contact with Dr G.M. Kelly as a name was in preparing for Honours Mathematics at the Leaving Certificate. The practice was to attempt all past papers. I still have the blue typeset papers for 1959 and 1960 which declare Professor T.G. Room as Chief Examiner and Dr G.M. Kelly as one of two Assessors.

In November 1960 Max married Imogen Datson.

Max's love of category theory also consolidated in the early 1960s while he was giving lectures on homology theory. In an attempt to understand the singular cohomology ring of a product of spaces, Max found that he could not even formulate the questions he wanted to ask without some basic concepts from category theory. Although he had heard of functors and natural transformations at Cambridge from the book by Eilenberg and Steenrod on *Foundations of Algebraic Topology*, he met Mac Lane's concept of categorical product first during lectures of Michael Atiyah at Harvard, while Max was a visitor at MIT in late 1962. Max had soon himself developed some lasting ideas in the field; one concept he called 'complex categories'. While visiting Tulane University (New Orleans) in 1963–1964, he met Eilenberg at Las Cruces giving a series of lectures on differential graded categories, which Eilenberg and Moore had recently invented; these were the same things as complex categories. Eilenberg insisted that Max remain in the US for another year. Indeed Eilenberg, on the spot, rang Alex Heller at Urbana and arranged a job at the University of Illinois for 1964–1965. Max's joint work with Sammy Eilenberg had germinated.

In January 1964, Max drove Imogen and two small children, Dominic and Martin, from New Orleans to Miami for a ten-minute talk at an American Mathematical Society meeting. Saunders Mac Lane introduced himself at the end of the talk and invited Max to visit Chicago. In the span of a couple of months, Max had met both founders of category theory: Eilenberg and Mac Lane. He soon met many more of the international categorical community and greatly valued these colleagues who became prominent in his life. Their tributes on his death attest to reciprocal affection.

My first contact with Max as a person was in 1965 when he taught two subjects to the Pure Mathematics Honours year at Sydney: category theory and topology. I found his lectures inspiring; they seduced me away from mathematical analysis. When Max had made a topic his own, he was able to provide a Bourbaki-style account of it at the drop of a hat. He actually arrived at our topology class prepared to teach us algebraic topology. After he asked us a few questions, it became clear we had done no topology, so he changed on the spot to teach us general topology. Soon he came to discuss product spaces and acted surprised that we knew nothing about the axiom of choice. Immediately Max listed six statements equivalent to the axiom of choice, explained them, and proceeded to prove the equivalence. He completed five of the implications in that one lecture, totally without notes and with only a few squats staring out the window taking stronger puffs at the cigar. The next lecture he finished the proof using a lemma I have not seen elsewhere. Max must have forgotten the precise form of the lemma since it was not the one we were asked to prove in the final exam. Brian Day and I became Max's postgraduate students in 1966, the year before he moved to the University of New South Wales as Professor of Pure Mathematics. Brian moved too, while I stayed at Sydney. Our relationship with our supervisor was very formal in those days.

Of course, by mid-1971, when I was at Macquarie and Max had returned to UNSW from Chicago, the formality had gone. Max had arranged a sabbatical at UNSW for the prominent category theorist Peter Freyd. During Freyd's stay Max organised, with the strong support of Bernhard Neumann, the first conference in Australia on category theory.

Max was elected Fellow of the Australian Academy of Science in 1972 and moved back to the University of Sydney as Professor in 1973. He was a true academic: erudite in the classics, prolific researcher and publisher, editor for several journals, successful department head, traveler, linguist, raconteur, and bon-vivant. He supervised five PhD students to completion; other supervisions include the MSc of Amnon Neeman in 1979. Max was very proud when, in 2002, Imogen gained her PhD in medieval drama.

Michael Makkai (McGill) claims Max as a logician in his passionate insistence on precision and clarity in mathematics and his belief in, and search for, the grand order at the heart of the world. Much of Max's work could be called higher order universal algebra.

He was very aware of how fortunate his life had been, and felt an obligation to give something back to the community. He was not motivated by making money, but by teaching and learning. To that end, he gave freely of his time to aspiring young mathematicians and to all those keen to learn. An example of this occurred when, frustrated by bureaucracies, he enlisted the power of the media and was able to borrow, for a blind girl in the Catholic school system, a mathematics textbook in Braille which had been gathering dust in a State Department of Education office. This commitment to social justice was further evidenced by his involvement with Action for World Development and his efforts to help the Aboriginal community in Redfern. He befriended Father Ted Kennedy, Mum Shirl and others active in these movements. He also questioned the morality of the Vietnam War, making himself quite unpopular with some of the clergy of the day.

Many were moved by the words of encouragement Max offered young category theorists in his speech at the 2006 Category Theory Conference dinner in Halifax, Canada.

Max had an active and analytical mind to the very end. He attended the Category Seminar at Macquarie two weeks before he died, excusing himself the next week because of an appointment. He started learning ancient Greek recently and in his last months was engaged in complex research on coherence theory, which he was typing despite failing eyesight. This research will be completed and published by collaborators in Canada and Italy.

Max Kelly is survived by Imogen, their children, Dominic, Martin, Catherine and Simon, and 10 grandchildren.

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A dense distance 1 excluding set in $\mathbb{R}^3$

M.S. Payne* and D. Coulson* **

Abstract

The study of the chromatic number of Euclidean space leads to the question of the maximum overall density that a single distance 1 excluding set can attain. In 1967 Croft presented a construction of what is still the densest known distance excluding set in $\mathbb{R}^2$. In this paper an analogous construction for $\mathbb{R}^3$ is presented which is the densest known distance excluding set in $\mathbb{R}^3$. A general approach for $\mathbb{R}^n$ is also discussed. The implications of the possibility that this is the densest set (or close to it) are discussed, particularly with regards to the lower bound for the chromatic number of $\mathbb{R}^3$ using measurable colourings.

Introduction

The chromatic number $\chi(\mathbb{R}^n)$ of Euclidean n-space is the minimum number of colours required to colour each point of the space such that no two points that are distance 1 apart receive the same colour. (We say the colouring *excludes distance 1*.)

The only known value is $\chi(\mathbb{R}^1) = 2$ as can be seen by $\mathbb{R} = \bigcup_{n\in\mathbb{Z}}[n, n+1)$ with the colouring

$$\text{colour}(x) = \begin{cases} 0 & \lfloor x \rfloor \text{ is odd,} \\ 1 & \lfloor x \rfloor \text{ is even,} \end{cases}$$

where $\lfloor x \rfloor$ is the greatest integer $\leq x$.

For other values of n only upper and lower bounds are known. The type of colouring can be restricted (e.g. to measurable sets (χ_m) or polygon tilings, etc.) and in many cases the lower bound can be improved under these restrictions [9], [13].

A related question is that of the greatest density ρ that a single colour can have in the space and be distance 1 excluding [12]. This question is of course restricted to colouring by measurable sets. Clearly $\chi_m \geq \rho^{-1}$. The set constructed in this paper gives a new lower bound for $\rho(\mathbb{R}^3)$.

A summary of some of the known bounds on χ, χ_m and ρ in low dimensions is given in Table 1.

The densest known set in $\mathbb{R}^2$ was constructed by Croft [3] and is depicted in Figure 1. It consists of a figure that is the intersection of a hexagon and a circle of slightly smaller diameter placed on the points of the equilateral triangle lattice. In other words, the figure

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Table 1.

$\mathbb{R}^2$	$\chi \geq 4$ [1]	$\chi_m \geq 5$ [4]	$\rho \leq \frac{12}{43}$ [11]	$\chi \leq 7^{(1)}$
$\mathbb{R}^3$	$\chi \geq 6$ [6]	$\chi_m \geq 6$ [4]	$\rho \leq \frac{7}{37}$ [10]	$\chi \leq 15$ [2]
$\mathbb{R}^4$	$\chi \geq 6$ [1]	$\chi_m \geq 7$ [4]	$\rho \leq \frac{4}{23}$ [5]	$\chi \leq 49^{(2)}$

(1) Demonstrated by a well known colouring based on regular hexagons.
(2) Demonstrated by a lattice/sublattice colouring in $\mathbb{R}^4$.

is a circle with six chords cutting off six equal segments. It is not difficult to confirm that in the optimal configuration the chords subtend an angle of 0.5266 (a transcendental multiple of π) radians at the centre and the density achieved is 0.2294 (thus $\rho(\mathbb{R}^2) \geq 0.2294$).

Heuristically we may imagine the process that leads to this construction as follows:

(1) Centre open circles of small radius on points distance 2 apart, arranged according to the equilateral triangle lattice.
(2) Increase their radius uniformly. We may do this until the circles have radius $\frac{1}{2}$ at which time 1 is the unique excluded distance.
(3) Continue to increase the area of the 'circles' by expanding towards the centres of the triangles but drawing the circle in from neighbouring circles in order to maintain an excluded distance.

The process is illustrated in Figure 1.

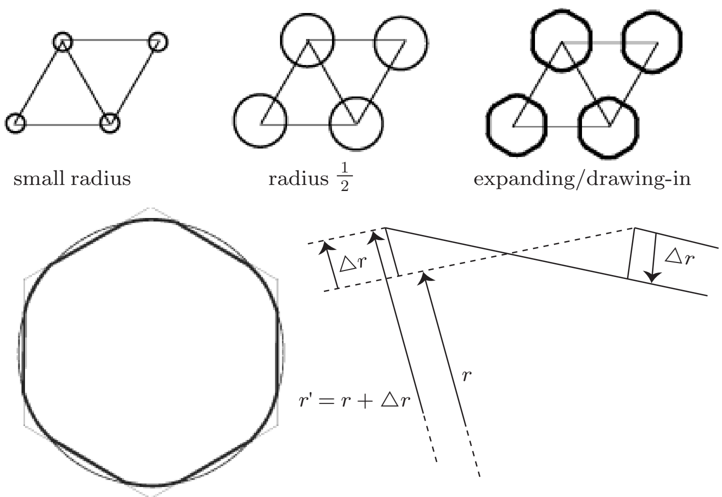

Figure 1. Top: the 'circle expansion' process with resultant hexagonal dice. Bottom: local change in a die with a change in the radius of the curved arcs, r (the 'curved' section of perimeter is dashed). The change in area will be (straight perimeter$-$curved perimeter)$\times\triangle r+O((\triangle r)^2)$.

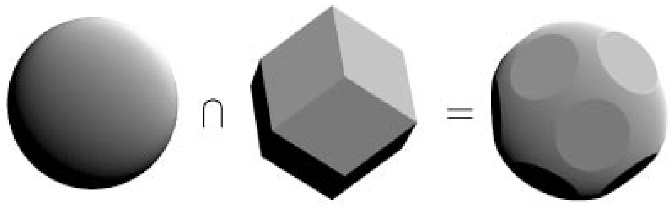

Figure 2. The Rhombic Dodecahedral Die of diameter $2r_s$, (RDD(r_s)) is the intersection of a sphere of radius r_s, $S(r_s)$ and scaled rhombic dodecahedron $(1-r_s)\mathcal{V}$.

We call the figures formed by this process 'hexagonal dice'. A neat way to characterise the dimensions of the hexagonal die of maximal area is to note that it occurs when the lengths of the chords and curved arcs are equal. To see this suppose that the hexagonal die has area A. If we increase the radius of the curved arcs by $\triangle r$ then the chords must be drawn in by $\triangle r$ to maintain an excluded distance. So

$$\triangle A = (\triangle r) \times L(\text{curved perimeter}) - (\triangle r) \times L(\text{straight perimeter}) + O((\triangle r)^2),$$

where $L(\ )$ is length. Thus the area is maximised when the lengths are equal.

This technique for creating a dense set can be generalised to $\mathbb{R}^n$ and the purpose of this paper is to present an analogous construction in $\mathbb{R}^3$. The general method is to start with the best known lattice based sphere packing for $\mathbb{R}^n$. By halving the radius of the n-spheres we can immediately produce quite a dense distance excluding set (with open spheres). The next step is to improve on this density by 'expanding' towards the lattice holes and 'drawing in' from the direction of neighbouring spheres. More precisely, we take the intersection of an n-sphere and a scaled Voronoi region (nearest neighbour region) of the lattice. The Voronoi region is a polytope with faces perpendicular to the lattice vectors. Hence the resulting figure is an n-sphere with flat faces sheared off, somewhat resembling a many-sided n-dimensional die. We will see below (as a check on calculations) that the equal curved and flat surface area characterisation holds for $\mathbb{R}^3$, and indeed in general $V_n(\text{die}) = \triangle r \times (V_{n-1}(\text{'curved part'}(\partial(\text{die}))) - V_{n-1}(\text{'flat part'}(\partial(\text{die})))) + O((\triangle r)^2)$ as the 'curved part' of the boundary is pushed out and the 'flat part is drawn in. Though this method can be applied to any underlying lattice it is not possible to give a general construction explicitly as each case depends on the packing lattice used.

The construction in $\mathbb{R}^3$

The approach is to use the face centred cubic lattice FCCL[1] (the best sphere packing lattice for $\mathbb{R}^3$) scaled so that distinct lattice points are at least 2 units apart, that is

$$\text{FCCL} = \{z_1(\sqrt{2},\sqrt{2},0) + z_2(0,\sqrt{2},\sqrt{2}) + z_3(\sqrt{2},0,\sqrt{2})\colon z_i \in \mathbb{Z}\}.$$

We centre open spheres of radius $\frac{1}{2}$ on these lattice vectors and 'expand' towards holes $r_s = \frac{1}{2} + \triangle$ and 'draw in' $r_d = \frac{1}{2} - \triangle$ from lattice vectors distance 2 away. This forms a rhombic dodecahedral die (RDD) that has $2r_s$ as an excluded distance. Here r_s and r_d are the maximum and minimum radii of the RDD respectively.

[1]Also known as $D_3 = \{\boldsymbol{v} \in \mathbb{Z}^3\colon v_1+v_2+v_3 \in 2\mathbb{Z}\}$ or $A_3 = \{\boldsymbol{w} \in \mathbb{Z}^4\colon w_1+w_2+w_3+w_4 = 0\}$.

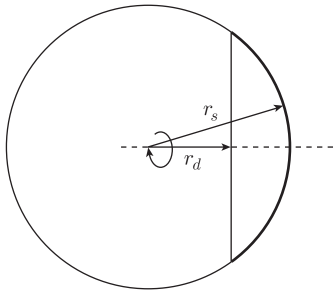

Figure 3. The 'caps' cut from the sphere $S(r_s)$.

More precisely the holes of FCCL are the vertices of the Voronoi region $\mathcal{V}$ of FCCL and their translates by vectors in FCCL. The Voronoi region $\mathcal{V}$ is a rhombic dodecahedron and is the convex hull of its vertices $\sqrt{2}\{\pm(1,0,0),\pm(0,1,0),\pm(0,0,1),(\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2})\}$ with volume $4\sqrt{2}$.

Maintaining $2r_s$ as an excluded distance where $2r_s$ is the diameter of the RDD means the RDD is the intersection of an open sphere of radius r_s and an open $r_d\mathcal{V}$ as pictured in Figure 2.

If $\frac{1}{2} \le r_s \le 4 - 2\sqrt{3}$ (so the sphere is inside the edges of the r_d scaled copy of $\mathcal{V}$) then the RDD will be a sphere with twelve identical circular 'caps' cut off by the faces of the rhombic dodecahedron $r_d\mathcal{V}$. This makes the calculation of its volume quite simple since the caps are just solids of rotation, see Figure 3.

$$\begin{aligned} V_{\text{cap}(r_s)} &= \pi \int_{r_d}^{r_s} r_s^2 - x^2 \, dx \\ &= \frac{\pi}{3}[4r_s^3 - 3r_s + 1] \quad \text{as } r_s + r_d = 1. \end{aligned}$$

Thus (for $\frac{1}{2} \le r_s \le 4 - 2\sqrt{3}$)

$$\begin{aligned} V_{\text{RDD}(r_s)} &= V_{S(r_s)} - 12 \times V_{\text{cap}(r_s)} \\ &= \frac{4\pi}{3}(9r_s - 11r_s^3 - 3) \end{aligned}$$

On the closed interval $\frac{1}{2} \le r_s \le 4 - 2\sqrt{3}$ the maximum volume of the RDD is

$$V^*_{\text{RDD}} = V_{\text{RDD}(\sqrt{3/11})} = \frac{4\pi}{3}\left(6\sqrt{\frac{3}{11}} - 3\right) \approx 0.5588,$$

given when $r_s = \sqrt{3/11}$.

Note also that for a rhombic dodecahedral die of (locally) maximal volume the 'flat surface area' will be equal to the 'curved surface area' (consider as in $\mathbb{R}^2$ referring to Figure 1, an infinitesimal change in r_s when volume is a local maximum). Clearly the proportion of 'curved surface area' to 'flat surface area' is a decreasing function of r_s thus there is a unique RDD giving a local maximum volume (when $r_s = \sqrt{3/11}$) and this must be the global maximum as $V_{\text{RDD}(0)} = V_{S(0)} = 0$, $V_{\text{RDD}(1)} = V_{(1-1)\mathcal{V}} = 0$ and $V_{\text{RDD}(r_s)}$ is a continuous function of r_s.

We now verify that when $r_s = \sqrt{3/11}$ for the RDD we have equality between 'curved surface area' and 'flat surface area'.

The surface area of the curved part of the cap is

$$\int_0^{2\pi} \int_0^{\arccos((1-r_s)/r_s)} r_s^2 \sin\phi \;\, d\phi \; d\theta = 2\pi(2r_s - 1)r_s.$$

Thus for $r_s = \sqrt{\frac{3}{11}}$,

$$\begin{aligned} \text{'flat area'} &= 12 \times \pi(r_s^2 - r_d^2) \\ &= 4\pi r_s^2 - 12 \times (2\pi(2r_s - 1)r_s) \\ &= \text{'curved area'}. \end{aligned}$$

The maximum volume for the rhombic dodecahedral die V^*_{RDD} gives a density of

$$\frac{V^*_{\text{RDD}}}{(V_{\mathcal{V}})} = \frac{\pi}{3\sqrt{2}}\left(6\sqrt{\frac{3}{11}} - 3\right) \approx 0.09878.$$

The density of the set of open spheres of radius $\frac{1}{2}$ placed on the same lattice will be $4\pi/3 \times 32\sqrt{2} \approx 0.09256$. Hence the density of the RDD based set is about 6.7% greater than that of the sphere based set.

The importance of this result

The best known general lower bound for the chromatic number of n-space using measurable sets is $\chi_m(\mathbb{R}^n) \geq n + 3$ as demonstrated by Falconer [4].

The hexagonal die constructed by Croft [3] in $\mathbb{R}^2$ has a density between $\frac{1}{4}$ and $\frac{1}{5}$. If this were the densest possible set, and therefore no set was as dense as $\frac{1}{4}$, it would give a second proof of the fact that $\chi_m(\mathbb{R}^2)$ is greater than or equal to 5.

The case in $\mathbb{R}^3$ is considerably more interesting. Falconer's result gives a lower bound of 6 for $\chi_m(\mathbb{R}^3)$, but the density of the construction presented here is between $\frac{1}{10}$ and $\frac{1}{11}$ (as is that of the plain sphere based set). If this were the densest possible set in $\mathbb{R}^3$ we would have a proof that $\chi_m(\mathbb{R}^3) > 10$ — quite an improvement.

Open questions

We have only considered sets based on a regular lattice, and only figures consisting of flat faces and spherical sections. Could there not be an irregular set which achieved a greater density? Certainly there could, but these sets are tremendously more difficult to analyse. Clues as to whether irregular sets might offer higher densities could be sought first in the two dimensional setting.

Acknowledgement

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Plane quadrilaterals

John Boris Miller*

Abstract

The various types of plane quadrilaterals are characterized by their side and diagonal lengths. Pantographs are described. The set of all congruence classes of quadrilaterals is a variety of degree six in E^6.

There are in Euclidean plane geometry some elegant theorems about cyclic quadrilaterals, most of them consequences of the angle properties in circles. For example, there is Ptolemy's theorem[1]: *In a cyclic quadrilateral, the product of the diagonals equals the sum of the products of opposite sides.* For noncyclic quadrilaterals, results are not so easily come by. One general theorem of note is Euler's[2]: *In any quadrilateral, the sum of the squares on the four sides is equal to the sum of the squares on the diagonals plus four times the square on the line joining the midpoints of the diagonals.* Euclid, in *The Elements*, devotes no more than passing interest in quadrilaterals which are not regular in some way, such as parallelograms. But later geometers have of course filled this gap, so that by the end of the nineteenth century the trigonometric properties of general quadrilaterals appear as suitable material for school textbooks, see [2], [1]. Nevertheless the emphasis was on cyclic quadrilaterals.

Plane quadrilaterals can be classified into *types* as follows[3] (see also Figure 1):

- *convex,* in which the two diagonals are internal and intersect;
- *nonconvex and not selfintersecting* ('dart'), in which one diagonal is internal and one external, the two not intersecting, the external diagonal spanning the concavity;
- *selfintersecting* ('zigzag'), in which one pair of opposite sides intersect, and both diagonals are external to the contained area and do not intersect;
- *(partially) degenerate*, in which a particular two adjacent sides lie in the same line (here there are three types as shown in Figure 1: a 'flag', where one vertex is an internal point of a side and two sides overlap, a 'triangle', where two adjacent sides are in one straight line but not overlapping, and a 'bent line', where two opposite vertices coincide);
- *fully degenerate*, in which the whole figure is contained in one dimension.

This classification results from the definition of a quadrilateral as a plane figure determined by four points and four line segments, each point being an endpoint of exactly two segments and each segment having each of its endpoints at a point. We shall not distinguish between

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[1] Ptolemy, Claudius, 2nd century AD. The converse also holds. See [3, pp. 225–227], where Lachlan's proof of the converse is also given.
[2] See [3, Vol. 2, pp. 401–402].
[3] Euclid, Book 1, contains a different classification in terms of degrees of symmetry, as squares, rhombuses, ...: see [3, Vol. 1, pp. 188, 189].

congruent quadrilaterals. The term 'convex' will conventionally be kept for the nondegenerate types. There are also several other degenerate types in which not all sidelengths are nonzero, but we will exclude these from consideration. So it is assumed hereafter that all sidelengths (but not necessarily diagonal lengths) are positive.

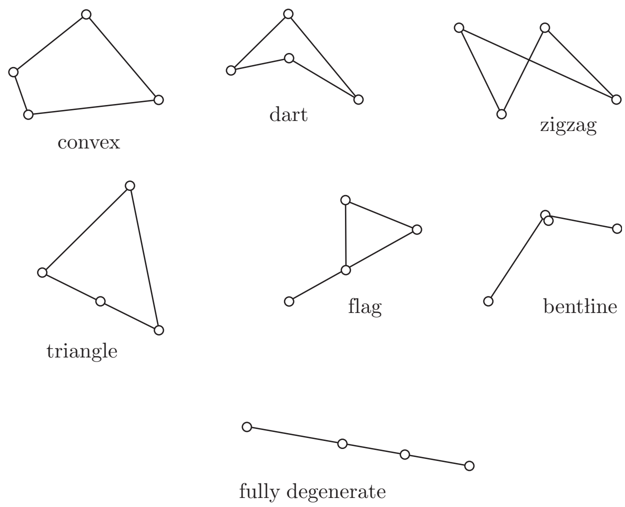

Figure 1. Types of plane quadrilaterals

It is easily proved that the four sidelengths and the two diagonal lengths together determine a quadrilateral uniquely. The four sidelengths and one diagonal length determine the quadrilateral to be one of two possibilities.

When can four given numbers be the side lengths of a quadrilateral? We find that: given an ordered tuple (a, b, c, d) of four positive numbers, a necessary and sufficient condition for the existence of a *convex* quadrilateral having those sidelengths, taken in that order, is:

$$\text{each number is less than the sum of the remaining three.} \tag{1}$$

For these inequalities clearly hold for the side lengths of any convex quadrilateral. Conversely, suppose a, b, c, d satisfy (1). Then

$$\max(a - b, b - a, c - d, d - c, 0) < \min(c + d, a + b),$$

so there exists a number x in this interval. The numbers a, b, x are the sidelengths of a nondegenerate triangle, so also are the numbers x, c, d, and these two triangles drawn back-to-back with common side x give a convex quadrilateral, one of whose diagonals has length x. Another solution, which is either a dart or a zigzag, results from drawing the triangles on the same side of side x.

The condition (1) is independent of the order of the tuple, so if there exists a convex quadrilateral with those side lengths in one order, there exist quadrilaterals for all other orders.

It is also true but a deeper result, using the intermediate value theorem, that (1) is necessary and sufficient for the existence of a convex cyclic quadrilateral with those sidelengths.

To see that (1) is also the appropriate condition for quadrilaterals of the other nondegenerate types, we introduce the notion of a *flexure*. This is a continuous operation which preserves the sidelengths of the quadrilateral and their order, but changes its shape. (It cannot in general be presented as a linear transformation of the plane in which the quadrilateral is embedded. We make the notion of continuity more precise presently.) With given sidelengths, the shape is a continuous function of an angle, say the angle between two chosen adjacent sides, or the angle between the diagonals, or of a diagonal length. By showing when each of the other forms can be reached from a convex form by flexing we can conclude that (1) is sufficient for that form (it is clearly necessary). If quadrilateral Ω can be flexed to Ω' we write $\Omega \sim \Omega'$.

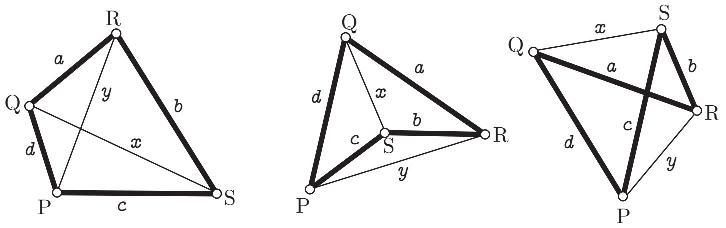

Figure 2. Notation for vertices and side and diagonal lengths

Therefore, let Ω be a convex quadrilateral with vertices P, Q, R, S and side lengths a, b, c, d, with $a =$ QR, etc. (see Figure 2, left). Suppose it is not a parallelogram: then there exists a pair of adjacent sides whose sum of lengths is less than the sum of lengths of the other pair of sides, say $a + b < c + d$. It can be flexed to a degenerate form which is a triangle Ω' say, in which the sides with lengths a and b are in the same line, meeting in the vertex R. Then a small further flexure to move R to the interior of Ω' is possible, giving a dart Ω''. Thus every quadrilateral which is not a parallelogram can be flexed to a dart.

To discuss flexure to a zigzag is less straightforward and depends upon a consideration of cases. Suppose $a > b > c > d$. We can fold side b onto side a to produce a flag Ω''' provided $c < (a - b) + d$. If

$$c \geqslant (a - b) + d, \tag{2}$$

then we try instead folding c onto b, which is possible provided $a < (b - c) + d$. If

$$a \geqslant (b - c) + d, \tag{3}$$

we try d onto c. If that does not work we try d onto a. Now the condition that all these four folds are impossible is a set of four inequalities; but these together lead to the contradiction $d \geqslant c$. Thus at least one of the folds is possible, and Ω is flexible to a flag. The case $a > b > c > d$ is one of 24 cases, which can be reduced to six needing separate consideration. After discussion of these, the end conclusion is given in Lemma 1.

Lemma 1. *Let Ω be a convex quadrilateral with sidelengths a, b, c, d in that order. Then Ω can be flexed to a flag in all cases except the following, in each of which Ω is flexible to a fully degenerate.*

(i) *Ω is a parallelogram.*
(ii) *The lengths of one pair of opposite sides are equal, and equal to the average of the lengths of the other two sides.*
(iii) *The lengths of one pair of adjacent sides are equal, and the lengths of the other pair are equal; Ω is a kite.*

Now if Ω can be flexed to a flag, then it can be flexed to a zigzag: for a small flexure of any flag to a zigzag is clearly possible, and flexure is an equivalence relation. Thus Theorem 1 follows.

Theorem 1. *The condition* (1) *holds for the sidelengths of any non fully-degenerate quadrilateral. It is a sufficient condition for the existence of a quadrilateral having those sidelengths, in that order, in the following classes:*

- $\mathbf{C}$*, the convex quadrilaterals,*
- $\mathbf{D} \cup \mathbf{P}$*, darts and parallelograms,*
- $\mathbf{Z} \cup (\sim \mathbf{Y})$*, zigzags and quadrilaterals which are flexible to fully degenerates.*

Here we have written $\mathbf{C}$ for the set of all convex quadrilaterals, $\mathbf{D}$ the darts, $\mathbf{P}$ the parallelograms, $\mathbf{Z}$ the zigzags, and $\mathbf{X}$ the set consisting of all degenerates, and $\mathbf{Y}$ the fully degenerates, with $\mathbf{Y} \subset \mathbf{X}$. Later we write $\mathbf{F}$ for the set of all flags, $\mathbf{T}$ the triangular degenerates. Note that $\mathbf{P} \subset (\sim \mathbf{Y})$.

Notation and topology

Given a quadrilateral Ω with sidelengths a, b, c, d in that order and diagonal lengths x, y, where x spans sides a, b and also c, d, we call

$$p = (a, b, c, d; x, y)$$

a *presentation* of Ω, and write $\Omega = \Phi(p)$ or sometimes $\Omega \equiv p$. We regard two quadrilaterals Ω, Ω' as equivalent and write $\Omega \equiv \Omega'$ if they are congruent figures, allowing congruence to include reversed orientation in the plane, and write $\mathbf{Q}$ for the set of all equivalence classes. Note that Ω has eight presentations $\sigma^j \tau^k p$ with $j = 0, 1, 2, 3$ and $k = 0, 1$, where σ and τ are the permutations

$$\sigma\colon (a, b, c, d; x, y) \mapsto (b, c, d, a; y, x) \text{ and } \tau\colon (a, b, c, d; x, y) \mapsto (d, c, b, a; x, y), \tag{4}$$

for which $\sigma^4 = \tau^2 = 1$, $\sigma\tau = \tau\sigma^3$, so that σ, τ are generators of a dihedral group. Thus $\Omega \equiv \Omega'$ means $\Omega \equiv p$, $\Omega' \equiv p'$ and $p = \sigma^j \tau^k p'$ for some j and k. Note the convention of listing in p the sides in order. It is necessary to introduce the notion of presentation because no satisfactory convention exists for all quadrilaterals to say with which side a listing of sidelengths should begin. (It is true that conventions can be invented for flags, for triangles and for darts, but that is not enough.) Write P for the set of all presentations.

Since p is a vector in Euclidean space $\mathrm{E}^4 \times \mathrm{E}^2$ we can use the Euclidean norm

$$\|\Omega\| = \|p\| = \sqrt{a^2 + b^2 + c^2 + d^2 + x^2 + y^2} \tag{5}$$

also as a function on quadrilaterals, since it is independent of the presentation. However, it is not then a norm, since the Euclidean distance does depend upon the presentations. To overcome this we introduce the function

$$D(\Omega, \Omega') = \min(\|p - \sigma^i \tau^j p'\|\colon i = 0, 1, 2, 3;\ j = 0, 1), \tag{6}$$

noting that $\|p - p'\| = \|\lambda p - \lambda p'\|$ for any permutation λ, so D is independent of the presentations used, and well defined.

Lemma 2. *D is a metric on* $\mathbf{Q}$.

Proof. It is clear that D is symmetric, and $D(\Omega,\Omega')=0$ when $\Omega\equiv\Omega'$. Suppose also $\Omega''\equiv p''$. For any h,i,j,k we have

$$D(\Omega,\Omega'')\le\|p-\sigma^h\tau^i p''\|\le\|p-\sigma^j\tau^k p'\|+\|\sigma^j\tau^k p'-\sigma^h\tau^i p''\|.$$

Fix j,k so that the first term on the right is $D(\Omega,\Omega')$, then choose h,i so that the second term is $D(\Omega',\Omega'')$. This proves that D satisfies the triangle inequality. □

In the same way we can introduce the metric $E(\Omega,\Omega')=\min(\|p-\sigma^i p'\|\colon i=0,1,2,3)$ on $\mathbf{Q}^\dagger$, the set of congruence classes where congruence is defined so as not to include reversed orientation. In $\mathbf{Q}^\dagger$ the further convention can be used for convex quadrilaterals, darts and partial degenerates that the order of listing sidelengths is clockwise, passing the inside of the quadrilateral on the right; but no such rule is possible for zigzags.

A flexure in $\mathbf{Q}$ can now be defined more precisely as a path $\Theta=\Phi\circ\phi$ determined by a continuous presentation-valued function $\phi\colon[0,1]\to P$ such that $\phi(t)=(a,b,c,d;x(t),y(t))$ where a, b, c and d are constants satisfying (1) and $x(t),y(t)$ satisfy (15), (16) below. Then $D(\Theta(t),\Theta(t'))$ is the minimum of five values $\|\phi(t)-\lambda\phi(t')\|$, where λ can be any of: the identity, $\sigma,\sigma^3,\tau\sigma,\tau\sigma^3$. Under the additional assumption that for all t the numbers $a,b,c,d,x(t),y(t)$ are all distinct we can show that λ must be the identity if $|\,t-t'|$ is small enough, that is, there exists $\delta>0$ such that

$$D(\Theta(t),\Theta(t'))=\|\phi(t)-\phi(t')\|=\sqrt{[x(t)-x(t')]^2+[y(t)-y(t')]^2}\quad\text{if}\quad|t-t'|<\delta. \tag{7}$$

This remark uses the uniform continuity of ϕ; see [6, Theorem 4.19, p. 91]. It asserts a locally minimizing property of ϕ.

Formulae

We mention some useful formulae for $\Omega\equiv p=(a,b,c,d;x,y)$. Let θ be the (acute) angle between the diagonals, and A the area. For convex quadrilaterals and darts the area is defined to be the area of the connected inside of the quadrilateral; for zigzags it is the modulus of the difference of the areas of the two bounded enclosed regions. Then in all cases we find that

$$A=\tfrac12 xy\sin\theta, \tag{8}$$

$$2xy\cos\theta=|a^2-b^2+c^2-d^2|, \tag{9}$$

$$16A^2+(a^2-b^2+c^2-d^2)^2=4x^2y^2, \tag{10}$$

$$2x^2y^2=\pm\sqrt{H(a,b,x)\cdot H(c,d,x)}-x^4+Sx^2+(a^2-b^2)(c^2-d^2), \tag{11}$$

where $S=a^2+b^2+c^2+d^2$ in (11), H denotes the Heron polynomial (see below), and the sign $+$ is to be taken if Ω is convex. The other cases are discussed presently. Proofs of identities (8), (9), (10) and many others can be found in [2, pp. 24–32], and [5, pp. 200–205], [1, pp. 168–176]. As a statement of a Pythagorean triple, (10) is particularly elegant. However, there is nothing to say that A or xy is rational when the side lengths are.

The Heron function H is the fourth degree symmetric polynomial in three variables $H(a,b,c)=-a^4-b^4-c^4+2b^2c^2+2c^2a^2+2a^2b^2$, which appears in the Heron formula $\sqrt{s(s-a)(s-b)(s-c)}$ for the area of a triangle with sides a,b,c and semiperimeter $s=\frac12(a+b+c)$. The area of the triangle is $\frac{1}{4}\sqrt{H(a,b,c)}$ (the formula is now attributed to Archimedes; see [4, p. 103]). Now in the case of a convex quadrilateral the area is that of the union of two triangles with common base x, so we can write

$$A = \tfrac{1}{4}\sqrt{H(a,b,x)} + \tfrac{1}{4}\sqrt{H(c,d,x)}.$$

Then substitution in (10) and simplification leads to (11). This gives a formula for y in terms of the other five lengths.

The equation (11) depends upon type as well as presentation. Let Ω be given, so that with presentation p we have (11) which we rewrite as $(11)_p$; there is also the equation

$$2x^2y^2 = \pm\sqrt{H(b,c,y)\cdot H(d,a,y)} - y^4 + Sy^2 + (b^2-c^2)(d^2-a^2); \qquad (11)_{\sigma p}$$

but $(11)_{\sigma^2 p}$ and $(11)_{\tau p}$ coincide with $(11)_p$ so there are only two distinct equations. Now the sign to be given the root term in $(11)_p$, which is the sign of

$$m(p) = 2x^2y^2 + x^4 - Sx^2 - (a^2-b^2)(c^2-d^2), \qquad (12)$$

is determined by whether the area of the quadrilateral is calculated as a sum or difference of areas. From this we arrive at the following rule for deciding the type of a quadrilateral, given in terms of its presentation p as opposed to an identifiable figure in the plane.

Theorem 2. *A quadrilateral Ω, given by one of its presentations p, is*

(i) *convex if $m(p)$, $m(\sigma p)$ are both positive,*
(ii) *a dart if $m(p)$, $m(\sigma p)$ are of opposite signs,*
(iii) *a zigzag if $m(p)$, $m(\sigma p)$ are both negative,*
(iv) *a flag if one of $m(p)$, $m(\sigma p)$ is zero and the other is negative,*
(v) *a triangle if one of $m(p)$, $m(\sigma p)$ is zero and the other is positive,*
(vi) *fully degenerate if $m(p)$ and $m(\sigma p)$ are both zero.*

A zigzag and a dart each has a distinguished pair of sides, say the intersecting pair for a zigzag, or the sides of the concavity for a dart. The following theorem characterizes these distinctions, using in one case the permutation function

$$\gamma\colon (a,b,c,d;x,y) \mapsto (a,x,c,y;b,d), \mathrm{P} \to \mathrm{P}. \qquad (13)$$

Theorem 3. *Let Ω be the quadrilateral $\Phi(a,b,c,d;x,y)$.*

(i) *If Ω is a dart and $m(p) < 0$ (so that $m(\sigma p) > 0$), then the edges of its concavity have lengths a, b (if $a+b < c+d$) or c, d (if $c+d < a+b$).*
(ii) *If Ω is a zigzag (so that $m(p) < 0$ and $m(\sigma p) < 0$), then $m(\gamma p)$ and $m(\sigma\gamma p)$ have the same sign; if it is negative then sides with lengths a, c intersect, if positive then sides with lengths b, d intersect.*
(iii) *If Ω is a triangle or flag, one diagonal is the sum or difference respectively of two adjacent sides, and this characterizes those features.*

Proof. (ii) and (iii) follow easily from the geometry. (i) Since $m(p) < 0$, the two areas $\frac{1}{4}\sqrt{H(a,b,x)}$, $\frac{1}{4}\sqrt{H(c,d,x)}$ are to be subtracted in $(11)_p$. This implies that x is the length of the external diagonal, so a, b or c, d are the lengths of the concavity edges. The concavity and diagonal are a triangle inside the other triangle, so the result follows from a theorem about triangles. □

Pantograph

If the midpoints of adjacent sides of Ω are joined in order, we get a parallelogram — the *median parallelogram* of the quadrilateral. Its existence is a welcome oasis of symmetry in an otherwise disorderly figure. The sides of the parallelogram are parallel to the diagonals of Ω, so its angle between adjacent sides equals the angle between the diagonals of Ω, the side lengths are half the diagonal lengths of Ω, and its area is half A (see Figure 3).

Now suppose instead that we are given an arbitrary parallelogram $\Lambda =$ FGHI, with sides of lengths f, g inclined at angle ω. Let QR be any line segment whose midpoint is F, then construct QP, RS having midpoints I, G respectively. We find that PS has H as its midpoint: so PQRS is a quadrilateral whose median parallelogram is Λ. Any parallelogram Λ generates in this way a doubly infinite family $\mathbf{Q}(\Lambda)$ of quadrilaterals for each of which it is the median. We call this family (with some abuse of terminology) the *pantograph generated by* Λ. The family contains quadrilaterals of all nondegenerate types as well as triangular and flag degenerates; all have the same diagonal lengths $x = 2f, y = 2g$, the diagonals being inclined at the same angle ω, have the same area, and therefore by (9) have the same value for $|a^2 - b^2 + c^2 - d^2|$. (It is perhaps more appropriate to regard a pantograph as a phenomenon in $\mathbf{Q}^\dagger$ rather than $\mathbf{Q}$.)

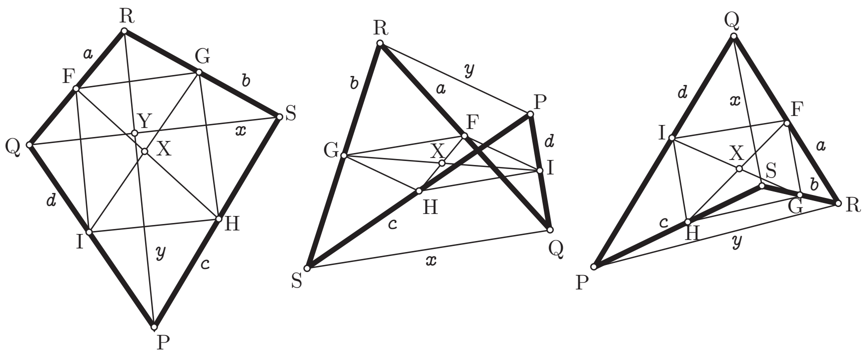

Figure 3. Quadrilaterals with their median parallelogram

If point P say traces out a figure Ψ, the other points Q, R, S trace out congruent figures, the figures of Q and S rotated through 180 degrees but with the same orientation. Here we see the action of a pantographic-like linkage. (More generally we can choose points F, G, H, I not as midpoints but as points of subdivision with specified ratio ρ, and obtain copies of Ψ with magnification ρ.)

If a and b are prescribed (subject to $a + b > 2f$) there are two quadrilaterals in $\mathbf{Q}(\Lambda)$ with those sidelengths for the sides through F and G, one of which is convex; so c and d are determined to that extent.

Theorem 4. *Let Λ be a parallelogram with sidelengths f, g and angle between the sides ω. Let Ω be a quadrilateral with diagonal lengths $2f, 2g$ and angle between the diagonals ω. Then $\Omega \in \mathbf{Q}(\Lambda)$.*

Proof. Construct Λ_1, the median parallelogram of Ω. Its sidelengths are f, g and the angles between its sides is ω. Thus $\Lambda_1 \equiv \Lambda$, and accordingly $\Omega \in \mathbf{Q}(\Lambda)$ since $\Omega \in \mathbf{Q}(\Lambda_1)$. (Recall that we identify congruent figures.) □

Corollary 1. *Given the parallelogram* Λ *as above, a quadrilateral* $\Omega \equiv (a, b, c, d; x, y)$ *belongs to* $\mathbf{Q}(\Lambda)$ *if and only if*

$$|a^2 - b^2 + c^2 - d^2| = 8fg\cos\omega, \qquad x = 2f, \qquad y = 2g. \tag{14}$$

The space Q

We now give a geometric description of the space $\mathbf{Q}$, treating a, b, c, d, x, y as coordinates in $\mathrm{E}^6 = \mathrm{E}^4 \times \mathrm{E}^2$, and start with P, the space of presentations. P is contained in the first 2^6-ant of E^6; the inequalities such as $a < b+c+d$ describe open half-spaces, so condition (1) confines P to a subset of an open wedge with edge-face the xy-plane and bounded by four primes. The conditions

$$\max(|a-b|, |c-d|) \le x \le \min(a+b, c+d), \tag{15}$$

$$\max(|b-c|, |d-a|) \le y \le \min(b+c, d+a), \tag{16}$$

which clearly must hold, describe the intersection of 12 further half-spaces. Altogether P is in the region bounded by $6+4+12 = 22$ primes. From these we can drop $x = 0$, $y = 0$ since these boundaries are implied by (15), (16), and thus count 20 primes. This set is mapped onto itself by each of σ and τ. Condition (11), when cleared of the root sign and simplified, shows that P is the variety V whose equation is

$$x^2y^4 + B(x)y^2 + C(x) = 0, \tag{17}$$

where

$$B(x) = x^4 - (a^2+b^2+c^2+d^2)x^2 - (a^2-b^2)(c^2-d^2), \tag{18}$$

$$C(x) = (b^2-c^2)(a^2-d^2)x^2 + (a^2c^2-b^2d^2)(a^2-b^2+c^2-d^2), \tag{19}$$

and is contained also in the variety V_σ got by applying σ to the equation for V. Note that the equation is invariant under τ. Thus P is the set obtained by intersecting the 20 half-spaces mentioned above with V, which is also their intersection with V_σ.

We now impose upon P the equivalence relation $\equiv$ where $p \equiv q$ if $p = \sigma^j\tau^k q$ for some $j = 0, 1, 2, 3; k = 0, 1$. $\mathbf{Q}$ is defined to be the set of equivalence classes $\mathrm{P}/\equiv$, with the metric topology induced by D. We leave it to the reader to verify Theorem 5.

Theorem 5. *The projection map* $\pi\colon \mathrm{P} \to \mathrm{P}/\equiv$ *(which coincides with the map* Φ *used earlier) is, with respect to the two metric topologies, both continuous and open, and therefore the metric topology of* D *on* $\mathbf{Q}$ *coincides with the quotient topology*[5].

These are indeed serendipitous outcomes. $\mathbf{Q}$ consists of the four sets $\mathbf{C}$, $\mathbf{D}$, $\mathbf{Z}$, and $\mathbf{X}$. Write C for the set $\pi^{-1}(\mathbf{C})$, etc. It is clear that the shape of any quadrilateral is preserved under magnification, angles being preserved: this is the phenomenon of proportionality, and means that sets C, D, Z, X are cones pointed at the origin.

The sets $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{Z}$ are open in $\mathbf{Q}$, the set $\mathbf{X}$ is closed, and contains $\mathbf{F}$ and $\mathbf{T}$. To prove the first three open we can argue from the geometry, or note that since the function m in (13) is continuous, the sets where m and $m \circ \sigma$ are of given signs are open, and invoke Theorem 2 and then the fact that π is an open map.

[5]See [7, Theorem 10.19, p. 62]. As a result about the projection map between metric spaces, this depends crucially upon (i) each equivalence class being finite, and (ii) the property here invoked as $\|\lambda p - \lambda p'\| = \|p - p'\|$ for all permutations λ.

To describe the connectivity of the space, write $\mathbf{Q}^* = \mathbf{Q} \setminus \mathbf{Y}$, where $\mathbf{Y}$ denotes as before the set of fully degenerates.

Theorem 6. *The sets* $\mathbf{C}$, $\mathbf{D}$, $\mathbf{Z}$, $\mathbf{F}$, $\mathbf{T}$ *are pathwise connected and hence connected subsets of* $\mathbf{Q}$.

Proof. We show that every Ω in $\mathbf{C}$ can be joined to the unit square by a path in $\mathbf{C}$. With Y being the point of intersection of the diagonals of Ω, let each vertex in turn be moved to the point on its diagonal at distance $1/\sqrt{2}$ from Y. Each such movement constitutes a path in $\mathbf{C}$, and their product path is therefore a path in $\mathbf{C}$ joining Ω to the unit square. This shows that $\mathbf{C}$ is pathwise connected. Similar proofs for the other sets can be constructed. □

We have, from Theorem 2 by similar arguments:

Theorem 7. *In the space* $\mathbf{Q}^*$ *with the relative topology,*

(i) $\mathrm{bdry}(\mathbf{C}) \cap \mathrm{bdry}(\mathbf{Z}) = \emptyset$,
(ii) $\mathbf{F} = \mathrm{bdry}(\mathbf{D}) \cap \mathrm{bdry}(\mathbf{Z}) \setminus \mathrm{bdry}(\mathbf{C})$,
(iii) $\mathbf{T} = \mathrm{bdry}(\mathbf{D}) \cap \mathrm{bdry}(\mathbf{C}) \setminus \mathrm{bdry}(\mathbf{Z})$,
(iv) *Every path joining* $\Omega \in \mathbf{Z}$ *to* $\Omega' \in \mathbf{C}$ *meets* $\mathbf{F}$*; every path joining* $\Omega \in \mathbf{Z}$ *to* $\Omega'' \in \mathbf{D}$ *meets* $\mathbf{F}$*; every path joining* $\Omega' \in \mathbf{C}$ *to* $\Omega'' \in \mathbf{D}$ *meets either* $\mathbf{F}$ *in at least two points, or* $\mathbf{T}$.

By Ptolemy's theorem and its converse, the cyclic quadrilaterals are represented by points on the intersection of P and the quadric $xy = ac + bd$. This intersection is a variety of dimension 4, degree 12.

For a given parallelogram $\Lambda \equiv (f, g, f, g;\ \mathrm{angle}\,\omega)$, the pantograph $\mathbf{Q}(\Lambda)$ is given by the intersection of P and the hypersurface (14), which is of dimension 3, degree 4. This intersection is thus of dimension 2, a surface of degree 12. The cyclic quadrilaterals in the pantograph therefore constitute a curve in E^6 of degree 24. It is a matter for further investigation, to discover how much of this curve is real, and whether the implied complexity is in fact a consequence of the representation.

Acknowledgments

I would like to thank Emanuel Strzelecki for useful comments, and Chris Hough for computational help. This is an abridged version of the original paper.

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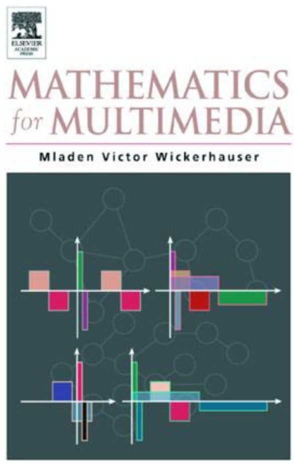
Book reviews

Mathematics for Multimedia

Mladen V. Wickerhauser
Academic Press 2004, ISBN: 0-12-748451-5

A good book should have good content, which has been carefully selected for the target audience and presented in an interesting and easy-to-read way. The content of this book is excellent. It covers main mathematical topics common and essential to multimedia. As a computer scientist working with a group of research students in the area of multimedia databases, it has been my view for some time that today's computer science graduates are not mathematically prepared to work in the area of signal processing, image processing and multimedia in general. So I studied this book with great interest. It did not take very long to disappoint me.

This is not a book I would recommend to a computer science PhD student who wants to work in multimedia research but is frustrated by many mathematical challenges involved in this area. This book is really about mathematics only, not about multimedia, not even attempting to show the links between the mathematical topics introduced and how they can be applied in multimedia.



One example is when discussing lossless encoding. One would expect an introduction to the famous Shannon's Information Theory, and some applications of entropy encoding in image compression. None of these are mentioned, not even in the suggested reading list. Another example is the part introducing DCT, which is used in JPEG encoding. This application is interesting to computer science students, and can motivate them to study this book and to understand the content better. Again, this book has discussed nothing more than DCT itself.

The publisher advertises this book as a book for 'computer science and multimedia students and professors'. It is not. It does not attempt to relate to the mathematical 'pearls' discussed in the book to the background and questions the targeted readers may have. Although it manages to spare a few pages to discuss binary representation of numbers and the definitions of graphs and trees, which are basic concepts all computer science students know very well.

Some algorithms in C are given in the book. As a computer science person, I would prefer to see some examples of the concepts and properties which sometimes I find difficult to understand, instead of some C code (not to mention different versions of the code using recursions or only iterations). After all, there are plenty of efficient libraries available. It is also interesting to comment on the first algorithm given in the book (Euclid's Algorithm). The code is correct, but quite difficult to understand, as one pre-condition is $a \geq b$, but the first iteration of the code is just to reverse this condition (by using $b\%a$). This algorithm can be represented in a much simpler way that is also easier to understand.

I recommend the book *Fundamentals of Multimedia* by Z.-N. Li and M.S. Drew (Pearson Education, 2004). *Fundamentals of Multimedia*, although not as concise and precise as *Mathematics for Multimedia*, is very easy for computer science students to read, and covers many topics discussed in this book.

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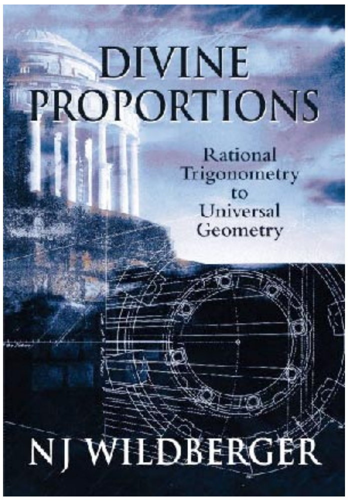
Divine Proportions:
Rational Trigonometry to Universal Geometry

N.J. Wildberger
Wild Egg, Kingsford, 2005, ISBN: 0-9757492-0-X 00-01 (51-01 97-01)

The book introduces an alternative approach to treating trigonometry and the Euclidean geometry of the plane. In the setting of the usual model of Euclidean geometry, the tools involved are not new, and the point is really a new emphasis. In particular at the core are *quadrance*, measuring the separation of points, and *spread*, measuring the separation of lines. In the usual setting and framework of Euclidean geometry, quadrance is exactly the square of the distance between points while spread agrees with the square of the sine of the angle between lines. The idea is that these measures should be taken as the basic building blocks of the theory, from both the conceptual and practical (i.e. calculational) point of view.

An observation made very early in the work is that the 'quadratic quantities' (to borrow a term from the author) of spread and quadrance may be used to develop a theory of trigonometric type calculations that avoid taking roots or using trigonometric functions. This is the 'rational trigonometry' from the title, and is so named because if calculations are handled suitably then they involve only rational expressions in the input number data. The latter point is central to the entire development since it means that many theorems may be given in a form (and are in this text) that is valid over an arbitrary number field (of characteristic other than two). The same idea is used to develop a version of aspects of Euclidean geometry that is valid over arbitrary number fields, and this is termed 'universal geometry'.

The first part of the book gives an overview of the rational trigonometry approach and its applications, and then in the next section this topic is developed more fully. Here, many of the classical elementary theorems are rephrased and recovered (but over a general field) using the new building blocks. For example, an old favourite from school days, the perpendicular bisector of a line, is seen, via a theorem, to be an 'equal quadrance to two points'; that is, all points on the bisector are of equal quadrance to the end points on the bisected line segment. Geometry is covered in the third part of the text, and here we find an algebraic and field independent treatment of a theory which generalises a significant part of the classical theory of triangles, polygons, circles, conics and tangents to arbitrary fields. The final part of the book deals with basic problems in a variety of directions ranging from projectile motion and surveying through to a discussion of rational replacements for spherical coordinates. While this section is called 'Applications', it really collects worked examples indicating how to treat a range of questions using the machinery of quadrance and spread.



As one might expect in a book which is claiming to challenge the traditional thinking, a large part of the work is concerned with justifying the theory. This is particularly true in the early sections where there is discussion pointing to claimed failings and inadequacies of past approaches and comparisons between the ideas in the text with what is asserted to be the traditional treatment. A worked example compares the approaches by calculating a certain section across a triangle in two ways. One treatment uses trigonometric functions and their inverses to get (via a calculator) an approximate solution while the other uses the rational techniques to obtain an exact solution in surd form. However as the author himself points out other classical approaches may be used that avoid calculating angles and will equally yield the exact solution. Indeed the rational trigonometry approach is just a collection of steps in Euclidean geometry! In the move to the language of 'Universal Geometry', some standard theorem statements become simpler while others become more complicated. This mostly occurs in an obvious way depending on whether the classical theorem is linear, quadratic or otherwise in nature. Among the most obvious, Pythagoras' theorem is a linear relation of quadrances. On the other hand, if the quarance between points A and B is written $Q(A, B)$, then in terms of rational trigonometry the collinearity of points A, B, and C is characterised by the so-called 'triple quad formula' $(Q(A, B) + Q(B, C) + Q(A, C))^2 = 2(Q(A, B)^2 + Q(B, C)^2 + Q(A, C)^2)$. This is hardly as intuitively appealing as the usual Euclidean notion of 'between': the point B is between A and C if and only if the distances between A and B, and between B and C add to the distance from A to C. On the other hand the symmetry in the triple quad formula means that we obtain an analogous result but where an ordering of the points is not needed.

Strictly speaking very little background is assumed of the reader and the material in the book is almost entirely elementary. Although most of the material could be treated over the reals the book would be confusing to a student not yet comfortable with the complex numbers. For example a 2-*proportion* is defined to be an expression $a : b$, for numbers a, b, with $a : b$ taken to be equal to $\lambda a : \lambda b$ for any non-zero λ in the field. Extending this notion in an obvious way, a line is defined to be 3-proportion $l = \langle a : b : c \rangle$ (with a, b not both zero), and this is said to be *null* if $a^2 + b^2 = 0$. In reality it seems unlikely that the book will appeal to the mathematically immature. For example, general fields (of characteristic other than two) are used throughout and there is significant emphasis on the applications to geometry over finite number fields, but fields are not defined formally. Also the simplifications that rational trigonometry brings to some calculations will mainly be evident to those who can appreciate the redundancies involved in the usual calculations via trigonometric functions and their inverses. To add to this viewpoint it should be pointed out that the style is extremely informal. Although there is a claim to develop Universal Geometry there is no definition of geometry in this text. Rather it is taken as a notion understood by the reader and what is presented in the section on Universal Geometry is simply a collection of theorems which are geometric in nature.

The cover of the book asserts 'This unique and revolutionary text establishes new foundations for trigonometric and Euclidean geometry'. There are more statements along those lines in the preface and introductory sections. Is there really a revolution within? The standard *model* of the Euclidean plane is the affine plane equipped with the standard quadratic form (the usual dot product). In this model, angle and length are not the basic building blocks but are derived quantities. If we make this our starting point then the move to the rational trigonometry approach is a gentle shift in emphasis rather than a revolution. This shift is concerned with how to deal with proofs, rather than with shaking foundations. I do not believe that teachers, students, engineers and other geometry consumers will be willing to discard the intuitive and, I would say, conceptually powerful notions of length and angle. For example, angle is not an arbitrary measure of the separation of lines but one which reflects the isotropy of the Euclidean plane. Experiments indicate that angular momentum is a conserved quantity. An upshot is that motions with constant angular velocity are common and important in our environment. The rotation of the earth is a salient example. It seems such motions will not yield a simple description in terms of spread.

These comments are not meant to imply that the book is without valuable ideas. A small shift can have major implications. Although the means to avoid calculating angles explicitly (for many calculations) is already available in Euclidean geometry, the text does provide a brief list of 'laws' and formulae which enable this to be done systematically. It makes sense to welcome these notions, these carefully packed parcels of information, into the Euclidean language. They may be manipulated in the same way as the classical laws. As indicated in the later sections this extension of the language enables many of the classical theorems of circles, triangles and so forth to be formulated in a universal and concise way that makes sense irrespective of the underlying field.

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A Generating Function Approach to the Enumeration of Matrices in Classical Groups Over Finite Fields

J. Fulman, P.M. Neumann and C.E. Praeger
AMS 2005, ISSN: 0065-9266, No. 830

This volume studies the proportions of cyclic, separable, semisimple, and regular matrices in a classical group defined over a finite field.

Let A be a matrix defined over a field F.

- A is *cyclic* if its characteristic polynomial $c(x)$ equals its minimal polynomial $m(x)$.
- A is *separable* if $c(x)$ has no repeated roots in the algebraic closure of F.
- A is *semisimple* if $m(x)$ is separable.
- If F is finite and A is an element of a classical group G defined over F, then A is *regular* if its centralizer in the corresponding algebraic group over the algebraic closure of F has dimension equal to the Lie rank of G.

The classical groups considered in the volume are $\mathrm{GL}(n,q)$, the group of all invertible $n \times n$ matrices with entries in $\mathrm{GF}(q)$; the symplectic group $\mathrm{Sp}(n,q)$, the subgroup of $\mathrm{GL}(n,q)$ consisting of all matrices which preserve a nondegenerate bilinear form; the orthogonal group $\mathrm{O}^{\epsilon}(n,q)$, the subgroup of $\mathrm{GL}(n,q)$ consisting of all matrices which preserve a nondegenerate quadratic form; and the unitary group $\mathrm{U}(n,q)$, the subgroup of $\mathrm{GL}(n,q^2)$ consisting of all matrices which preserve a nondegenerate sesquilinear form. These families of matrix groups and their related simple quotients are fundamental objects of study.

Let A be an element of a classical group defined over a finite field F. A cyclic matrix A has the property that the vector space of $1 \times n$ matrices over F is cyclic as an $F\langle A\rangle$-module. In almost all cases, A is cyclic if and only if it is regular; A is semisimple if and only if it has order coprime to the characteristic of F; and A is separable if and only if it is regular and semisimple.

This volume complements and extends earlier work. Fulman [2] and Wall [9] independently used generating functions to study the proportion of cyclic and separable matrices in $\mathrm{GL}(n,q)$. They proved that, as $n \to \infty$, the limiting proportions of cyclic and separable matrices in $\mathrm{GL}(n,q)$ are $(1-q^{-5})/(1+q^{-3})$ and $1-q^{-1}$ respectively.

Neumann & Praeger [6] obtained estimates for the proportion of cyclic and separable matrices in $\mathrm{M}(n,q)$, the set of all matrices with entries in $\mathrm{GF}(q)$. In [7] they extended these results for cyclic matrices to the general classical groups, which preserve the appropriate form up to scalar multiple. Guralnick & Lübeck [5] have also determined the proportions of separable matrices in classical groups and the exceptional families of groups of Lie type. Their combinatorial and geometric approach is very different and their results are useful only for fields of size at least 5.

The authors extend the results of Fulman and Wall on cyclic and separable matrices in $\mathrm{GL}(n,q)$ to the other classical groups and also obtain similar results for the classes of regular and semisimple elements. For each classical group G of dimension n defined over $\mathrm{GF}(q)$, and for each of the four named types of matrices, they obtain very precise estimates for the probability $p(n,q)$ that a random element of G is of this kind. They focus on the case where q is fixed and n is allowed to grow. In almost all cases, $p(n,q)$ has the form $1-aq^{-1}+b(n)q^{-2}$ or $1-aq^{-3}+b(n)q^{-4}$, where a is a constant depending on the type of the matrix and the classical group, and $b(n)$ depends on n but is bounded above and below independently of n for sufficiently large n. Observe that, for fixed n, these probabilities tend to 1 as $q \to \infty$. They prove the existence of the limit $p(\infty,q)$ as $n \to \infty$, obtain precise expressions for this limit as a function of q, find sharp lower and upper bounds, and estimate its rate of convergence. The results are useful for all finite fields.

The central components studied are power series generating functions expressed as infinite products, and characteristic and minimal polynomials for elements of classical groups. The results of Fulman and Wall demonstrate that the limiting probabilities for cyclic and separable matrices in $\mathrm{GL}(n, q)$ are rational functions of q. It is not yet known which, if any, of the remaining limits have this property.

Recently, Britnell (see, for example, [1]) has extended these techniques to some of the other classical groups not covered here, obtaining similar results.

An important motivation for this work is the use of such elements in the design and analysis of algorithms for the study of subgroups of $\mathrm{GL}(n, q)$. For example, Neumann & Praeger [8] have exploited cyclic matrices to obtain a variation of the MeatAxe, a fundamental algorithm to decide if an FG-module fixes any proper subspace of the underlying vector space. Other applications include the study of fixed point free permutations (derangements) in finite transitive permutation groups [3] and monodromy groups of curves [4].

A major strength of this volume is its precision. The techniques are powerful, providing detailed insight into the structure of the classical groups. The volume is very well written and its material is both clearly presented and summarized. It contains a wealth of important results and represents a major contribution to our understanding of these groups.

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AMSI News

Philip Broadbridge*

Being positive within a harsh reality

This is the last opportunity I have to write before the release of the 2007 Federal Budget. Therefore I will make some comments on the state of the mathematical sciences. I have formed an impression by visiting many AMSI member institutions.

Notwithstanding the serious effects of severe drought, shortages in water supply and possible ongoing induced climate modification, we have recently experienced a time of relative national prosperity. We have the opportunity to invest in public infrastructure and services that will shore up our future wellbeing. Unfortunately, for the mathematical sciences, lean times are already upon us. I am aware of four university mathematics departments that downsized last year. Four other universities have drafted letters of redundancy to mathematics staff this year.

In the May 2006 issue of the Gazette, Grant Cairns said, ‘I do think Mathematics is in trouble’. After a full investigation, The National Strategic Review of Mathematical Sciences Research, reporting in December 2006, noted, ‘The nation’s capacity to support research, research training and advanced education in mathematics and statistics is diminishing rapidly’ (http://www.review.ms.unimelb.edu.au).

It is all too easy to dismiss the authors as being negative. Yet individually each of them has made very positive contributions to research and education. They deserve to be heard. Future generations of young Australians deserve the opportunity to participate in a vibrant intellectual environment. Those of us who have been well rewarded for our mathematical achievements yet who have failed in our obligations to the system beyond our own little empire, are culpable if that system decays.

The report of the Strategic Review contains specific recommendations to reverse the downward spiral. I urge you to read the report and to alert your political representatives to those recommendations that will require deliberate yet modest expenditure within a favourable budget.

Mathematical research is part of a long cultural tradition of intellectual inquiry. In an ideal advanced civilisation, that tradition should be well supported. In reality, material investment is stimulated by the promise of material returns. Investment in the mathematical sciences is a very positive step towards social harmony and material prosperity by way of enhanced capability in problem-solving, analytical reasoning, rational decision-making and product design. AMSI is working positively to build bridges to other disciplines and to industry. This is a way of broadening public support. For example, many employees from

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the energy sector have registered for the workshop and short course, 'Mathematics of Electricity Supply and Pricing'. In 2008, we hope to run a similar event on mathematics of water resources, involving not only AMSI and ICE-EM (International Centre of Excellence for Education in Mathematics) but also the International Centre of Excellence in Water Resources Management.

We are fortunate to have seen the simultaneous operations of AMSI, ICE-EM and MASCOS (Centre of Excellence for Mathematical and Statistics of Complex Systems). Similar centres can be supported in the future provided we continue to communicate forcefully with members of the broad community and with its elected political leadership.

Director of AMSI since 2005, Phil Broadbridge was previously a professor of applied mathematics for 14 years, including a total of eight years as department chair at University of Wollongong and at University of Delaware.
His PhD is in mathematical physics (University of Adelaide). He has an unusually broad range of research interests, including mathematical physics, applied nonlinear partial differential equations, hydrology, heat and mass transport, and population genetics. He has published two books and more than 80 refereed papers, including one with 147 ISI citations. He is a member of the editorial boards of three journals and one book series.

News

General News

CSIRO: The Big Day In — A CSIRO Vacation Program

A couple of months ago in Sydney, 80 undergraduates participated in 'The Big Day In' to give them a closer look at where a research career might take them.

The Big Day In (http://www.cmis.csiro.au/bdi6/index.htm), held on 15–16 February, was the culmination of the 10th year of the Undergraduate Vacation Scholarship Program of three CSIRO divisions (Mathematical and Information Sciences, Industrial Physics and the ICT Centre). These divisions joined forces to offer an 8–12 week research project over the summer uni vacation. The students they attracted are enrolled in mathematics, computer science, physics, statistics, medical science, finance, engineering and related disciplines.

Bigger and better

Instigator and host of the event, CMIS Chief Dr Murray Cameron (http://www.csiro.au/people/psok.html), said that each year the event has improved with students from 21 universities participating in 2007.

'We seem to be attracting stronger students, giving them better projects and getting better presentations from the students. There was a terrific buzz around the meeting and the supervisors seemed to be getting real value from the students' efforts. Guiding a vacation student is a pleasure, not a chore,' he said.

Dr Cameron also commented that, given the current controversy around shortages (http://www.csiro.au/news/ps2lr.html) of maths and science skills in Australia, 'we are particularly pleased to be able to reinforce students' enthusiasm for building a full time career as maths and science professionals'.

This year there were 40 CSIRO projects available in a variety of places in Australia working with senior scientists across many science and engineering disciplines. Students were paid $550 per week as well as receiving training in presentation skills and free travel and accommodation for The Big Day In, held this year in Sydney.

The Big Day In involved not just the CSIRO vacation students, but also about 40 other students from AMSI's Summer School. Over a two day program, all students presented the results of their research to an audience of fellow students, supervisors, CSIRO scientists and other experts in their fields.

The lighter side of science

As well as giving their own presentations, the students participated in a panel called 'So you want to be a scientist?' hosted by Chris Krishna-Pillay, CSIRO educator and comedian. Chris also ran a session on how to find your perfect match, in which two students found true love the scientific way. He also moderated a comedy debate, held for the first time in 2007, with CMIS scientists as the debaters.

The debate topic was: 'On average, statisticians are sexier than mathematicians'. Needless to say, much creative evidence was brought to bear on the subject, with the result being judged by which team got the loudest wolf whistles from the audience at the end. The negative team won by a few decibels.

This was Chris K-P's second Big Day In, an event he sees as a fantastic way to add value to CSIRO vacation scholarships.

'It's engaging, informative and fun. This is such a powerful way to reach out to tomorrow's researchers. Bringing them together under the CSIRO "umbrella" provides a memorable experience and shows that CSIRO is committed to the future', Chris said.

'Hands on' maths

This year, as in the past, the students' presentations were of a very high standard. For their part, the students gave a very positive view of the experience. They gave the highest ratings to supervisors' guidance and involvement, as well as challenging and rewarding projects. Benefits were 'hands on' experience, skill building, relationship building, making career contacts, experience in a self-sufficient environment, exposure to real life work protocols and presentation.

CSIRO has a long history of offering vacation scholarships across its divisions. The scholarships provide students with the opportunity to work on a real-world problem and to develop future links with CSIRO. Many previous vacation students have maintained a relationship with CSIRO in either Honours projects, PhDs, and eventual CSIRO employment. The scholarships are open to students who have completed at least three years of undergraduate study and who have maintained a credit average or better.

Griffith University:

Griffith University is currently undertaking a review of its undergraduate mathematics offerings within the Science, Environment, Engineering and Technology Group (SEET) under the direction of Professor Ned Pankhurst, PVC (SEET) and Mr David Edwards, Dean of Learning and Teaching. The review is chaired by Professor Peter Healy, School of Biomolecular and Physical Sciences, Griffith University. The review aims to: (i) provide Undergraduate and Postgraduate pathways to the mathematical sciences profession; (ii) provide a range of attractive courses meeting the mathematics and statistics needs of the Group's programs; (iii) enhance the research base both within the mathematical sciences discipline and in interdisciplinary areas.

The review has raised the possibility of having a central 'Maths@Griffith' Group to streamlining course offerings while providing better paths for students and provide greater opportunity to establish a research focus for mathematics at Griffith University.

Monash University:

Book publications: Burkard Polster, The Shoelace Book: A Mathematical Guide to the Best (and Worst) Ways to Lace Your Shoes, American Mathematical Society, 2006.

Completed PhDs

Monash University:

- Dr Andrew Marshall, *The Madden–Julian Oscillation: Role of air-sea interaction and the MJO-ENSO relationship*, supervisor: Professor Michael Reeder.
- Dr Daniel Tokarev, *Galton–Watson processes and extinction in population systems*, supervisor: Professor Fima Klebenar.

University of Melbourne:

Dr Will James' details were listed incorrectly in a previous issue of the *Gazette*.

- Dr Will James, *The enumeration of heaps and almost-convex polygons*, supervisors: Professor Tony Guttmann, Associate Professor Aleks Owczarek, Dr Richard Brak.

University of New South Wales (Australian Defence Force Academy):

- Dr James Caunce, *Mathematical modelling a wool scour bowl*, supervisors: Dr Steven Barry and Dr Geoff Mercer.

University of Queensland:

- Dr Joshua Ross, *Density dependent Markov population processes: models and methodology*, supervisor: Professor Phil Pollett.

University of South Australia:

Dr Manju Agrawal's details were listed incorrectly in a previous issue of the *Gazette*.

- Dr Manju Agrawal, *Dynamics and control of drug user populations*, supervisor: Dr Yalcin Kaya.

Appointments, retirements and promotions

Monash University:

- Associate Professor Hans Lausch has retired.
- Dr Pam Norton has taken early retirement.
- Dr Malcolm Clarke has taken early retirement.

University of Ballarat:

- Professor Gary Bloom, Mathematics Department, City University of New York, is spending three months (March–May 2007) of his sabbatical at the University of Ballarat, working with Joe Ryan and Mirka Miller.
- Professor Costas Iliopoulos, Kings College, London, has won a Royal Society grant for collaborative research with people at the University of Ballarat. He plans to visit Ballarat during November/December 2007.

University of Melbourne:

- Dr Mei Zhang has been apppointed Research Fellow (MASCOS)
- Ms Annabelle Lopez has been apppointed Research Assistant (AMSI)

- Dr Jingyu Shi has been apppointed Postdoctoral Research Fellow
- Dr Andre Costa, Research Fellow, has left the University
- Dr Charles Lilley, Research Fellow, has left the University

University of New South Wales:

- Colin Rogers, Professor of Applied Mathematics and Head of Department, is retiring on the first of July, and has had the honour of Emeritus bestowed on him.

University of Queensland:

- Tony Roberts has a continuing appointment at Level C at the Mathematics Department. Tony's research focuses on modelling the structure and properties of heterogeneous materials.
- Dr Geoffrey J. Goodhill has a level D continuing joint appointment between the Queensland Brain Institute and the School of Physical Sciences. Dr Goodhill's lab uses theoretical, computational and experimental techniques to investigate how biological nervous systems become wired up during development.
- Dr Joseph Grotowski has a Senior lecturer continuing appointment in Mathematics. Dr Grotowski's research area is geometric analysis and geometric evolution equations.
- Dr Tian Tianhai has accepted a three-year Lord Kelvin Fellowship in the Department of Mathematics, the University of Glasgow, followed by a tenured level B lectureship in the area of computational biology and bioinformatics.

University of Sydney:

- Associate Professor Don Taylor retired on 30 March. Professor Nalini Joshi has taken over from him as Head of School.

University of Western Australia:

- Dr Des Hill has been promoted to Senior Lecturer.

Awards and other achievements

- Congratulations to AustMS member Professor Alan Welsh of the Australian National University, who has been elected a Fellow of the Australian Academy of Science. For the full list, see http://www.science.org.au/media/newfel2007.htm .
- Adjunct Professor Phill Schultz has been awarded the Chancellor's Medal for services to mathematics education in Western Australia.
- Professor Cheryl Praeger has been awarded an ARC Australian Professorial Fellowship.
- Dr Michael Giudici has been awarded an ARC Australian Research Fellowship.
- Professor Matthew England has been awarded an ARC Federation Fellowship, which he took up in March 2006. Not more than 25 of these are awarded each year. The Fellowship runs for five years, and is funded at over $1.5 million. The goal of the Fellowship is to explore Australian climate extremes and their associated predictability.

  Professor England has been joined by Professor Andrew Pitman from Macquarie University. They will be co-directors of the UNSW Climate Change Research Centre. The

Centre will seek to train the next generation of climate scientists with a special emphasis on quantitative skills. The Centre will be offering a Science major in 'Climate' from 2008. Further details on the Centre are at:
http://www.science.unsw.edu.au/news/2007/climatecentre.html

The story of Professor England's Fellowship can be found at:
http://www.unsw.edu.au/news/pad/articles/2005/jun/Fed_Fellows.html .

Conferences and Courses

On 9 May, CMIS is running a one-day course in Melbourne on 'The Statistics of Extremes in Climate Change'. The presenter is Rick Katz from the US National Center for Atmospheric Research.

Web: http://www.csiro.au/events/StatisticsofExtremes.html

2007 Winter School in Mathematical and Computational Biology

The ARC Centre in Bioinformatics and the Institute for Molecular Biosciences (UQ) will be hosting the 4th annual Winter School in Mathematical and Computational Biology for the week of 25–29 June 2007. This year's Winter School is designed to introduce mathematical and computational biology and bioinformatics to postgraduate and advanced undergraduates, postdoctoral researchers, and others working in the fields of mathematics, statistics, computer science, information technology, engineering, and the biological, chemical and medical sciences.

The Winter School is structured to present one topic per day, this year's being: bio-image analysis, quantification and classification; modelling and simulation of cellular processes; prediction and modelling of protein structure and dynamics; statistical analysis of gene expression; and computational neuroscience. The speakers, many of whom are leading national and international authorities, have been selected for their ability to make their topic accessible to and exciting for a non-specialist audience.

Registration is now open.

Web: http://bioinformatics.org.au/ws07

Symmetries and Stability Workshop

This workshop takes place on 26–29 June 2007, and will be held at the University of Canberra. The meeting seeks to bring together mathematicians working in group theoretic methods in the study of differential equations including the qualitative theory of ordinary and partial differential equations, bifurcation theory, symmetries, stability and dynamics of pattern formation as well as symmetry reduction broadly understood. Invited speakers include Bjorn Sandstede, Claudia Wulff (Univ. of Surrey), George Bluman (Univ. of British Columbia) and Georg Gottwald (Univ. of Sydney). Students particularly are encouraged to attend, and there will be poster sessions and contributed talks.

For more information please go to http://www.amsi.org.au/Symmetries.php

Conference announcement and call for papers — EMAC 2007

The 8th Biennial Engineering Mathematics and Applications Conference will take place at University of Tasmania, Hobart, 1–4 July 2007.

EMAC 2007 is organised by the Engineering Mathematics Group, a special interest group within the Australian and New Zealand Industrial & Applied Mathematics Division of the Australian Mathematical Society. The meeting is held biennially in alternate years to CTAC (Computational Techniques and Applications Conference). EMAC 2007 is hosted jointly by School of Mathematics and Physics (University of Tasmania), School of Engineering (University of Tasmania), Department of Maritime Engineering (Australian Maritime College) and Tasmanian Partnership for Advanced Computing (University of Tasmania).

Contributed papers are currently being sought in the following areas: the development and use of mathematical and statistical methods in engineering and applied mathematics; mathematical modelling of engineering, technical and industrial applications; mathematics education in engineering, applied mathematics and related disciplines.

Key dates: abstracts due 2 April 2007; notification of acceptance 30 April 2007; early-bird registration 7 May 2007; presenters' registration 22 June 2007.

Full written papers will be due at the conference and published in the ANZIAM electronic journal subject to peer review.

Registration: full $550, students $330, or $500/$280 if paid before 7 May. Registration includes welcome reception, lunches, morning and afternoon teas and conference dinner.

Web: http://www.maths.utas.edu.au/emac2007

GL07 Geometry and Lie Theory:
A conference marking Gus Lehrer's 60th birthday

First week: 2–6 July 2007, Australian National University, Canberra
Second week: 9–13 July 2007, University of Sydney, NSW
Organisers: James Borger, Peter Bouwknegt, Anthony Henderson, Bob Howlett, Amnon Neeman and Andrew Mathas.

Web: http://www.maths.usyd.edu.au/u/SemConf/lehrerfest.html

ICE-EM Australian Graduate School in Mathematics

2–20 July, University of Queensland, Brisbane, Australia.

Postgraduate students and early career researchers from mathematics, statistics and cognate disciplines are invited to apply for a place in the 2007 Graduate School. Distinguished international researchers will present two courses in each of three streams:

Statistics: Professor Anthony Davison, Ecole Polytechnique Federal de Lausanne, Geneva and Professor Louise Ryan Harvard School of Public Health, USA (this stream runs for two weeks only, 2–13 July).

Algebra and Combinatorics: Professor Alexander Pott, Otto-von-Guericke-University Magdeburg, Germany and Professor Nick Wormald, University of Waterloo, Canada.

Lie Theory: Professors Leticia Barchini and Roger Zierau, Oklahoma State University, USA and Professor Nolan Wallach, University of California, USA.

Places are strictly limited to 25 per stream and acceptance will be based on academic merit. Generous subsidies, covering up to 100% of travel and accommodation costs, are available to students living outside the Brisbane metropolitan area, with first preference given to students from institutions affiliated with AMSI and PRIMA and to students and staff from universities in Asia.

Closing date for second round applications is 20 May 2007 and registration forms are available on the website www.maths.uq.edu.au/IAGSM/. For further details contact Lynda Flower at l.flower@sps.uq.edu.au.

Joint Australia–China Meeting on Non-linear Partial Differential Equations And Related Topics

The Australia–China joint meeting on 'Non-linear Partial Differential Equations and Related Topics' will be held at the Oaks Calypso Plaza Suites Coolangatta, Gold Coast (near Brisbane) from 2 to 6 July 2007. The conference is to host a number of leading Chinese and Australian mathematicians as well as providing the opportunity for young researchers and PhD students to get together to present new work in the field. This conference will enhance the strong relationship between China and Australia in this area. Nonlinear PDE has traditionally been one of the major Chinese scientific research areas and over the years there have been many interactions with Australian researchers in this area.

The Conference organising committee is: Neil Trudinger, (Chair, The Australian National University); Joseph Grotowski (University of Queensland); Min-Chun Hong (University of Queensland); Bevan Thompson (University of Queensland); Xujia Wang (The Australian National University). The conference is sponsored by AMSI .

Web: http://www.maths.uq.edu.au/ hong/conference.html

Call for Papers: 8th International Conference on Finite Fields and Applications (Fq8)

9–13 July 2007, Melbourne, Australia

The aim of this conference is to bring together researchers from all aspects of finite fields, theory, computation and applications. Previous meetings have been in Las Vegas (USA), Glasgow (Scotland), Waterloo (Canada), Augsburg (Germany), Oaxaca (Mexico) and Toulouse (France). The conference is organised by Deakin University.

Web: http://fq8.it.deakin.edu.au

SOHO 19/GONG 2007 — Seismology of Magnetic Activity

9–13 July 2007, Monash University

Web: http://www.soho19.org/

Combinatorial Design and Graph Theory Workshops

In 2007, The Centre for Discrete Mathematics and Computing at the University of Queensland will hold a series of three Combinatorial Design and Graph Theory Workshops.

The first workshop was scheduled for April 2007, with invited speakers including Professors Mike Grannell and Terry Griggs from the Open University UK, Dr Nicholas Cavenagh from the University of New South Wales, Dr Ian Wanless, Monash University and Dr Michael Hoffmann, Leicester University UK.

The second workshop is scheduled for the last week of July with invited speakers including Professor Chris Rodger from Auburn University, USA, Dr Sule Yazici from Koc University, Turkey and Dr Abdollah Khodkar from the University of West Georgia, USA.

The third workshop is scheduled for the last week of November with invited speakers including Professor Ebad Mahmoodian from Sharif University of Technology, Iran and Prof Ralph Stanton, Manitoba University.

Web: http://www.maths.uq.edu.au/∼carloh/workshops2007/

For additional information please contact Diane Donovan at dmd@maths.uq.edu.au

The 2007 AustMS Annual Conference

The 51st Annual Meeting of the Australian Mathematical Society will take place at La Trobe University, Melbourne, 25–28 September 2007.

Please go to the website for information on accommodation options including a link to a booking page for Rydge's on Bell. With regard to accommodation, please be aware that (i) we are still seeking cheaper options to suit students and (ii) the AFL Grand Final takes place the day after the meeting finishes which will inevitably place pressure on accommodation availability throughout Melbourne (so book early).

The conference will also host special sessions that cover a broad range of topics in mathematical research and its applications. Each special session will have a keynote speaker and suggestions for these speakers can be made directly to the session organisers once the session details are announced in the near future.

Contact: John Banks, J.Banks@latrobe.edu.au

Web: http://www.latrobe.edu.au/mathstats/maths/conferences/AMS2007/

2007 International Symposium on Computational Models for Life Sciences

CMLS'07 will be held 10–12 December, 2007, Gold Coast, Queensland, Australia.

With an emphasis on the applications of computer, physical, engineering and mathematical models for solving modern challenging problems in life sciences, CMLS'07 aims to bring biologists, medical and health-science researchers together with computational scientists to focus on problems at the frontier of computational life sciences. Symposium Co-Chairs: Tuan D. Pham and Xiaobo Zhou

Web: http://www.it.jcu.edu.au/∼pham/CMLS07/CMLS07.htm

CATS 2008 — Computing: The Australasian Theory Symposium

The 14th Computing: The Australasian Theory Symposium (CATS) will be held at the University of Wollongong, New South Wales, Australia, during January 22-25, 2008. CATS is one of the two premier annual conferences in theoretical computer science in the Asia-Pacific (http://uob-community.ballarat.edu.au/ pmanyem/cats-08/submission.html).

Authors are invited to submit papers that present original and unpublished research on topics including (but not limited to) the following areas: algorithms and data structures, complexity theory, graph theory, graph algorithms and combinatorics, semantics of programming languages, algorithms on strings, optimisation, formal program specification and transformation, computational algebra and geometry, computational biology, logic and type systems, and new paradigms of computation.

Submitted papers will be thoroughly refereed and accepted papers will appear in the electronic proceedings at http://crpit.com. For full details on the publication policy, go to http://crpit.com/Authors.html .

More information: http://uob-community.ballarat.edu.au/ pmanyem/cats-08/

Program Committee Co-Chairs:
James Harland, RMIT University, Australia (Bjames.harland@rmit.edu.au);
Prabhu Manyem, University of Ballarat, Australia (p.manyem@ballarat.edu.au).

Web: http://uob-community.ballarat.edu.au/ pmanyem/cats-08/

Visiting mathematicians

Visitors are listed in the order of the last date of their visit and details of each visitor are presented in the following format: name of visitor; home institution; dates of visit; principal field of interest; principal host institution; contact for enquiries.

Dr Maria Pilar Gil Pons; Polytechnical University of Catalonia, Spain; until 5 May 2007; –; MNU; Professor John Lattanzio

Prof Gennian Ge; Zhejiang University, China; 21 April 2007 to 07 May 2007; –; UWA; Prof Cheryl Praeger and A/Prof Caiheng Li

Dr Henrik Latter; Cambridge; 7 February 2007 to 7 May 2007; astrophysical fluid dynamics; USN; D.J. Ivers

Mr Sergei Haller; Justus-Liebig-Universität, Gießen; 16 January 2006 to 11 May 2007; algorithmic methods for lie groups; USN; S. Murray

A/Prof Nader Tajvidi; Lund Institute of Technology, Sweden; 12 Feb 2007 to 13 May 2007; –; UMB; –

Mr Henrik Baarnhielm; Queen Mary College, London; 1 April 2007 to 16 May 2007; group theory algorithms; USN; J.J. Cannon

Mr Jonas Rasmussen; University of Copenhagen; 23 February 2007 to 16 May 2007; algebraic number theory; USN; D.R. Kohel

Dr Alice Devillers; Universite Libre de Bruxelles, Belgium; 13 March 2007 to 18 May 2007; –; UWA; Prof Cheryl Praeger

Prof Fazia Maaouia; University of Tunisia; until 18 May 2007; –; MNU; Prof Fima Klebenar

Dr Ruth Baker; University of Oxford; 4 December 2006 to 25 May 2007; –; UMB; –

Dr Alex Kitaev; Steklov Mathematical Institute; 24 January 2007 to 25 May 2007; singularities and other properties of integrable systems; USN; N. Joshi

Prof Charles Leedham-Green; Auckland; 15 April 2007 to 26 May 2007; computational group theory; USN; J.J. Cannon

Prof Eamonn O'Brien; Auckland; 15 April 2007 to 26 May 2007; computational aspects of group theory; USN; J.J. Cannon

Matthew Turner; University of Exeter, UK; 8 May 2007 to 27 May 2007; –; UWA; Prof Andrew Bassom

Serge Kruk; Oakland University; 16 January 2007 to 30 May 2007; –; UMB; –

Stephen Glasby; Central Washington University; late February to June 2007; –; UWA; Cheryl Praeger

Dr Andrey Radostin; Russian Academy of Sciences; 1 March 2007 to 1 June 2007; –; UWA; Dr Elena Pasternak

Dr Willem de Graaf; Universita Di Trento; 9 April 2007 to 8 June 2007; computational group theory; USN; J.J. Cannon

Dr John Voight; Minnesota; 1 to 19 June 2007; arithmetic geometry; USN; J.J. Cannon

Mr Constantin Caranica; –; 12 May 2007 to 30 June 2007; computational aspects of modular forms; USN; J.J. Cannon

Prof Phil Howlett; University of South Australia; 8 January 2007 to 30 June 2007; –; UMB

Dr Yan Wang; University of South Australia; 15 January 2007 to 30 June 2007; –; UMB; –

Dr Peter Brooksbank; University of Oregon; 14 June 2007 to 2 July 2007; computational group theory; USN; J.J. Cannon

Prof Jean Michel; University of Paris VII; 2 to 7 July 2007; geometry and lie theory; USN; A. Henderson

Prof Anthony Davison; Ecole Polytechnique Federal de Lausanne, Geneva; 2 to 13 July 2007; modelling of statistical extremes, higher order asymptotics, and statistical modelling, particularly for applications in biology; UQ; Tony Bracken

Professor Louise Ryan; Harvard School of Public Health, USA; 2 to 13 July 2007; statistical methods related to environmental risk assessment for cancer, developmental and reproductive toxicity, and other non-cancer endpoints; UQ; Tony Bracken

Professor Cedric Bonnafe; University of Science and Technology Besancon; 1 to 14 July 2007; geometry and lie theory; USN; A. Henderson

Professor Corrado De Concini; University of Rome; 8 to 14 July 2007; geometry and lie theory; USN; A. Henderson

Prof Tonny Springer; Utrecht; 2 to 14 July 2007; geometry and lie theory; USN; G.I. Lehrer

Dr Emmanuel Letellier; Concordia University; 6 to 16 July 2007; geometry and lie theory; USN; A. Henderson

Prof Leticia Barchini; Oklahoma State University, USA; 2 to 20 July 2007; representation theory of real reductive lie groups; UQ; Tony Bracken

Prof Alexander Pott; Otto-von-Guericke-University Magdeburg, Germany; 2 to 20 July 2007; coding theory and cryptography (binary sequences, Boolean functions); UQ; Tony Bracken

Prof Nolan Wallach; University of California; 2 to 20 July 2007; methods and applications of invariant theory; UQ; Tony Bracken

Prof Nick Wormald; University of Waterloo, Canada; 2 to 20 July 2007; combinatorics, graph theory and random graphs; UQ; Tony Bracken

Profs Roger Zierau; Oklahoma State University, USA; 2 to 20 July 2007; representation theory of real Lie groups and related geometry; UQ; Tony Bracken

Prof Claudio Processi; University of Rome; 8 to 21 July 2007; geometry and lie theory; USN; A. Henderson

Prof Francois Digne; Universite de Picardie Jules-Verne; 25 June 2007 to 29 July 2007; geometry and lie theory; USN; A. Henderson

Inessa Epstein; University of California; 15 January 2007 to 31 July 2007; –; UMB; –

Prof Buyung-Moo Kim; Chungju National University; 31 July 2006 to 31 July 2007; integral theory; USN; D.E. Taylor

Dr Toshio Ohnishi; Institute for Statistical Mathematics, Tokyo, Japan; 5 February 2007 to 31 July 2007; –; USQ; Dr Peter Dunn

Prof Xianhua Li; Suzhou University, China; 10 March 2007 to 10 August 2007; –; UWA; A/Prof Caiheng Li

Mr Mikael Johansson; Mathematisches Institut, Fakultät für Mathematik und Informatik, Jena, Germany; 10 September 2007 to 20 October 2007; computational aspects of group cohomology; USN; J.J. Cannon

Dr Eric Badel; INRA (National French National Institute for Agricultural Research): Wood Material Laboratory (LERMAB) – Nancy, France; February to November 2007; –; QUT

Prof Robert Liebler; Colorado State University, USA; 1 October 2007 to 11 November 2007; –; UWA; Prof Cheryl Praeger

Ms Weiwei Ren; Yunnan University, China; February to Dececember 2007; –; UWA; A/Prof Caiheng Li

Jan Saxl; Cambridge University; mid-November to December 2007; –; UWA; Cheryl Praeger

Mr Mohamad-Reza Mohebbi; Tehran University of Medical Sciences, Iran; 18 March 2007 to 1 February 2008; –; UMB; –

Dr Youyun Li; Hunan Changsha University; 1 May 2006 to 1 May 2008; –; UWA; A/Prof Song Wang

Dr Alireza Nematollani; University of Shiraz; 15 December 2007 to 15 December 2008; multivariate analysis and time series; USN; N.C. Weber

Dr M. Iranmanesh; Yazd University, Iran; 10 June 2007 to 10 March 2008; –; UWA; Prof Cheryl Praeger

The Australian Mathematical Society

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Secretary:	Dr E.J. Billington	Department of Mathematics University of Queensland QLD 4072, Australia. ejb@maths.uq.edu.au
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Business Manager:	Ms May Truong	Department of Mathematics Australian National University ACT 0200, Australia. office@austms.org.au

Membership and Correspondence

Applications for membership, notices of change of address or title or position, members' subscriptions, correspondence related to accounts, correspondence about the distribution of the Society's publications, and orders for back numbers, should be sent to the Treasurer. All other correspondence should be sent to the Secretary. Membership rates and other details can be found at the Society web site: http://www.austms.org.au.

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Queensland Univ. Techn.:	G. Pettet	Univ. Canterbury:	C. Price
RMIT Univ.:	Y. Ding	Univ. Waikato:	W. Moors

Publications

The Journal of the Australian Mathematical Society

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The ANZIAM Journal

Editor: Prof. C.E.M. Pearce
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Bulletin of the Australian Mathematical Society

Editor: Dr A.S. Jones
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The *Bulletin of the Australian Mathematical Society* aims at quick publication of original research in all branches of mathematics. Two volumes of three numbers are published annually.

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