Book reviews

The Mathematics Compendium

A.C. Fischer-Cripps
Institute of Physics Publishing 2004
ISBN: 0-7503-1020-0

In the Preface to his *Mathematics Compendium*, Fischer-Cripps (Fischer-Cripps Laboratories Pty Ltd, NSW) writes that "It is not surprising that [...] the pass rate for students in science courses in mathematics subjects is usually much lower than that for their other subjects." After reading this book I am left with the impression that Fischer-Cripps himself was among this group of under-performing students in mathematics. Indeed, this is arguably one of the worst mathematics books ever written. Worse still, it exactly aims at those students with limited confidence in their own mathematics skills, promising them an "alternative (read easier) viewpoint of mathematics", but delivering chaos and misery instead.

Apart from being very sloppy and often incomplete — even the very first table of formulae on page 2 contains a typo, subtraction is misspelled as substraction, the general equation of a straight line is given as $y = mx+b$, the anti derivative of $1/(x+a)$ is stated as $\log(x + a) + C$, a formula is given for the angle between lines of equal slopes, and so on — the book contains some seriously bizarre "mathematics". Take for example the discussion of the natural logarithm on page 37. The author introduces the function $F(x)$ by $F'(x) = 1/x$ so that the reader will recognize $F(x) = \log|x|+C$. Then a list of properties of $F(x)$ is stated. Among these are the beauties

$$F'(kx) - F'(x) = 0 \tag{1}$$

and

$$F'(kx - x) = 0. \tag{2}$$

Clearly, Fischer-Cripps is as confused about rules of differentiation as some of our weaker first year calculus students. In (1) he obviously tries to say that

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}F(kx) &= kF'(kx)\\ &= k\cdot\frac{1}{kx} = \frac{1}{x} = \frac{\mathrm{d}}{\mathrm{d}x}F(x),\end{aligned}$$

but evidently he had no access to a good mathematics compendium to remind him of the chain rule .... What he has in mind with (2) remains a mystery. Hopefully it is ridiculous enough to alert even the most unsuspecting beginner to the dangers of this book.

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Insurance Risk and Ruin

David Dickson
Cambridge University Press 2005
ISBN: 0-521-84640-4

As a teacher of undergraduate actuarial students in insurance risk theory for the past decade, I have for some time been painfully aware of the relative lack of appropriate level texts in this area. So, even if it had no other redeeming features, Professor Dickson's *Insurance Risk and Ruin* would be laudable for helping to alleviate this notable gap in the educational literature. Fortunately, Professor Dickson's text is much more than just a placeholder in the lexicon of actuarial science textbooks. It provides an accessible treatment of some of the most fundamental concepts in the classical development of insurance portfolio modelling, admirably treading the tightrope between rigorous detail and intuitive argument. As with any text which attempts to present mathematical material without dwelling on the exacting technical details, it occasionally strays in one direction or the other. For the most part, however, the text traverses the critical topics from compound distributions and convolutions in aggregate claims modelling to compound Poisson processes and ruin probabilities at a level which is neither so detailed as to bog down the reader, nor so vague as to inhibit real understanding of the underlying theory.

The structure of the book moves in logical progression from dealing with distributions for individual claims through distributions for aggregations of claims in a fixed period to distributions for processes of claim distributions through time. The initial sections on individual claim distributions covers all the common choices of positively skewed distributions as well as providing a useful grounding in the application of statistical principles to actuarial and insurance frameworks, including discussions of reinsurance, policy excess and utility theory. However, the one notable omission in coverage for the entire text is here in these sections, with the lack of a discussion of the use and development of mixture distributions (as distinct from mixed distributions, which are discussed in the context of reinsurance and policy excess). Of course, no table of contents will ever map perfectly to the ideals of every reader, but some discussion of this topic would have made for a useful grounding when discussing the comparisons of the collective risk and individual risk models, as well as combining compound Poisson insurance portfolios in general. The subsequent sections gradually build up more complex model structures and provide a standard, but eminently understandable, presentation of compound distributions and convolutions in a very practical and useful way. Finally, the last sections of the book traverse the more complex area of compound Poisson processes, breaking the material into chapters, the first focussing on discrete time processes and the latter on continuous time. The ordering here is the common one, though it is sometimes suggested that the reverse order of presentation is actually more pedagogically sound. In any event, both chapters compliment each other as necessary components to a full understanding of the material, as well as a solid grounding for the material in the final chapters. These latter chapters include a selection of advanced topics including the 'old standards' of severity of ruin and optimal reinsurance structures. One might have hoped for some lead-in to a more advanced course on the topic with a brief discussion of measure theory and martingales, but then again, this would have likely exceeded the appropriate level of the text, which it maintains admirably throughout, and allows the book to stand as a complete course on its own.

The style of the presentation is readable and straightforward, if not overly innovative; though given the lack of texts in this area noted previously, the text's very existence could be seen as important innovation. A structured and consistent mathematical notation is employed throughout, making for ease of reference across the topics of the text; however, a notational glossary page would not have gone astray. The book's fundamental distinction is, as is noted in the author's introduction, the liberal inclusion of discussions of recursive calculation techniques. These are a clear favorite of the author, and while one can quibble as to the relative number of pages that ought to be afforded to such topics, it does give the book its unique signature. From this reviewers perspective, modern computing technology and highly accurate approximation techniques based on, for instance, saddlepoint methods are a preferable approach to these issues, and some comparison of alternatives might bolster the relevance of these sections. Nevertheless, every book needs its unique character, and the recursive calculation sections are this book's emblem. In addition, the book also contains a broad array of useful exercises and, perhaps more importantly, model solutions.

On the whole, Professor Dickson's text gives an excellent undergraduate treatment of an important and often daunting subject and will make a more than useful addition to the shelves of both actuarial educators and students alike.

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Alan Turing's Automatic Computing Engine

B. Jack Copeland (Ed.)
Oxford University Press 2005
ISBN: 0-1985-6593-3

> "Solving one's difficulties by thought rather than equipment."

25% or so of this book is immensely valuable. Turing's original document: "Proposed electronic calculator" is reproduced in full, including circuit and other diagrams. Also included are notes based on lectures by Turing and Wilkinson in 1947, and a wonderful lecture by Wilkinson from 1974 (first published in *The Radio and Electronic Engineer*, **45** (1975), 336–340). The quotation that begins this review is from that lecture.

Turing's proposal is an object lesson to present day writers of grant proposals: a seductively simple but powerful introduction, followed by increasingly precise and technical details. The budget is disarmingly simple (but perhaps more explanation might be needed today). He asks for £11,200 or about $.5M in today's currency. Turing succeeded in getting his project approved.

Unfortunately the blurb on the dust jacket (and also the subtitle: *The Master Codebreaker's Struggle to Build the Modern Computer*) of this overly expensive book ($185) leaves OUP open to a possible suit for misleading advertising. It is not true to say that the book "describes Turing's struggle to build the modern computer." Nor is the rest of the paragraph justified by the book. What the book *does* do is to provide documents on Turing's progress.

There *are* the makings of a book here, but what there is needs to be organized logically, probably largely historically, but certainly not with basic technical details commencing on p.341. Incidentally the clearest descriptions of Turing's coding are to be found in Turing's original proposal and not in the new text. The present editing is minimal, and a number of the contributors are deceased, so rewriting the book would be onerous rather than profitable.

On the history of computers, apart from the pieces already noted, the reader is urged to consult Martin Davis's *The Universal Computer: The Road from Leibniz to Turing*, W. W. Norton, New York and London, 2000. This is a work by an American written with honesty and without bias but unfortunately not referenced in the book under review. (Davis's name appears once in a reference to another work of his and not in the index.) In particular, Davis looks at the achievements of Turing and von Neumann objectively and clearly.

Even from a bad book there are lessons to be learnt. In this case there are quite a number: The importance of mathematicians and engineers working together rather than one sending instructions to the other; the balance between speed and legibility in algorithms and computer programs; the importance of proper management; and the value of practical mathematics, to mention but a few.

For the reader of the Gazette the change in emphasis required for doing numerical analysis rather than pure analysis is well brought out in the short piece by J. G. Hayes *Programming the Pilot ACE* pp. 215–222. As I observed at first hand in Britain in the sixties, Leslie Fox and others had an unnecessarily hard struggle for the recognition of work on numerical analysis. This reflects ill on the closed-mindedness of the mathematical community.

Finally, as Whiteside, the editor of another giant English mathematician's works, told this then young historian of mathematics many years ago: "Look at the originals." There is no better advice. Copeland has made some of Turing's unpublished works accessible. For this he is to be thanked.

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The Pea and the Sun
A Mathematical Paradox

Leonard Wapner
AK Peters 2005
ISBN: 1-56881-213-2

Popular science books play an important role in the early stages of the careers of many budding research scientists. Stephen Hawking's *A Brief History of Time* and Brian Greene's *The Elegant Universe*, along with a large number of similarly outstanding popularisations of physics, have surely influenced the interests of many high school and undergraduate students. The list of authors who have succeeded in making physics accessible to the interested general reader goes on: Penrose, Rees, Thorne, Feynman, even Einstein have written successful popularisations of physical theories. Similar success attends many of the authors, such as Stephen Jay Gould and Richard Dawkins, who seek to explain biology, and particularly evolution, to the layperson.

The popularisation of pure mathematics is somewhat more problematic. While one may get the feeling from reading a Stephen Hawking book that one knows what a black hole 'is' (whether or not that feeling is justified), it is not possible to get such a feeling for many of the theorems of pure mathematics. Some books circumvent this problem by concentrating on the 'human interest' element of the history of the relevant area — thus the life story of Évariste Galois makes perhaps more appearances in popular mathematics books than one might consider are warranted. Simon Singh has been particularly successful in this area (human interest in general, not life stories of Galois, although he is pretty good at them too). He has also had the knack of picking an area of mathematics to write about that is comprehensible to the reader without much background. Fermat's last theorem, for example, while involving a very technical proof, is one of the most easily-stated difficult problems that exist, and it was the subject of one of Singh's most successful books. Others (such as Martin Gardner, whom Leonard Wapner acknowledges as a major influence on his choice of career as mathematician) focus more on 'recreational' mathematics, allowing technicalities to be cast aside and giving the reader a taste for the excitement of mathematics without the tedium of familiarisation with definitions and preliminary lemmas and such. Mathematics has been blessed with such maestros of this most difficult of arts as Singh, Gardner, Raymond Smullyan and Ian Stewart.

Leonard Wapner, in *The Pea and the Sun*, adopts a combination of these approaches. Short biographies of many of the fascinating mathematicians involved in the development of the ideas leading up to the Banach-Tarski theorem are related. Many delightful recreational morsels relevant to the main theorem are also presented, including 'scissors congruence', some elegant dissection problems and various paradoxes (the latter are used to illustrate the exact sense in which the Banach-Tarski paradox is a 'paradox').

Where Wapner diverges from the canonical popular maths book types is in his decision to present, in detail, the proof of the Banach-Tarski theorem. The Banach-Tarski theorem states that a three-dimensional sphere may be divided into a finite number of pieces which can then be put back together to make two spheres of the same size as the original (if one assumes the Axiom of Choice). It is commonly referred to as a paradox because it appears that volume has been 'created'. This is resolved as soon as one accepts that the intermediate pieces are not measurable, i.e. their volume can not be defined. This is a technical and subtle theorem for the general reader — Wapner's goal is a very ambitious one! The proof presented is complete, and is certainly very accessible to any later-year undergraduate student of maths. This is a feat in itself, and makes the book excellent reading for anyone in this position. It is doubtful, however, whether the proof would be accessible to the "general public", who Wapner appears to have in mind. The concept of a group of 3×3 matrices is essential to the proof, and it is tricky to pick up without a course at university level.

That certainly does not mean that the book is not worth reading for the interested high school student or layperson. The biographical and recreational sections are excellent fodder for the youthful mathematical imagination.

However, the later chapter entitled 'The Real World' is unfortunate. It certainly ought not to have been included, and readers are advised to skip it. It focusses on the relevance of the Banach-Tarski theorem to physics, and the connections made might be described as tenuous at best. It is suggested that a 'Banach-Tarski mechanism' is responsible for the inflation of space-time after the big bang, that the Banach-Tarski theorem may be the "source of all randomness and chaos in the world", and that various fundamental particle reactions in which quarks are created may be accounted for by "Banach-Tarski duplication". Wapner has clearly not researched the credibility of his sources for this chapter, and some are highly questionable. Propagating such poor science devalues the rest of the book.

Excluding this chapter, *The Pea and the Sun* is a very good book for reasonably experienced mathematics students, and worth reading for what is surely the most accessible explanation of the proof and the implications of the Banach-Tarski theorem that exists. For the majority of the intended audience, certain parts of the book will be fascinating, but the proof that is the keystone of the book, binding the ideas presented in earlier chapters, will be inaccessible. For this reason, the book probably does not 'work' for as large an audience as intended. This is exactly why the popularization of pure mathematics is so very difficult — almost all serious theorems do not lend themselves to popular exposition. Ultimately, successful books about pure mathematics fall into the historical or the recreational categories. Intrepidly stepping outside these boundaries, Wapner has done as good a job of making the Banach-Tarski theorem accessible as it seems possible to do (with the exception of his dalliance with physics). Unfortunately, Wapner's considerable skill and imagination in rendering the technical material understandable are not quite enough to bring the book into the realm of 'popularisation'.

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Advanced Engineering Mathematics with Maple

R.J. Lopez
Maplesoft
ISBN: 1-894511-86-7

If your university teaches engineers maths at 2nd year undergraduate and upwards, and uses Maple in teaching, you really should look at this CD, published by maplesoft A list of its contents can be viewed at http://www.maplesoft.com/. by selecting 'products' and then selecting 'eBooks'.

Several years ago, I reviewed, in this Gazette a *book* and CD-ROM by the same author, and with essentially the same title. (See *AustMS Gazette* **28** (2001), 160–162.) The earlier book and CD was published by Addison-Wesley.

There are obvious problems with large books as paper and printing is expensive. In my earlier review, there was a plea to consider just publishing via CD. The present title does precisely this. For material which depends on a software package, an additional advantage is that the CD can be kept up-to-date with the releases of the software. The present CD is for maple 10.01. It runs trouble-free. I'm impressed by the consistent style. I didn't discover any hyperlinks which didn't work, so the attention to detail is to be commended. I liked the fact that hyperlinks to the "Table of Contents" and to the "Index" were always available at the bottom of the "page/screen". I wonder if it might be worthwhile to make more use of hyperlinking?

(i) Perhaps the index should send one to where the word or phrase occurs rather than merely to the section in which it occurs.
(ii) More use of hyperlinking when refering back to relevant earlier parts might be useful.

For universities using Maple in their teaching of engineering mathematics this CD is thoroughly worthwhile. (UWA engineers use Matlab, not Maple, so UWA will not be adopting the CD as a 'text'.)

I've already had a visitor (Francis Benyah) from a university where Maple is used in teaching saying that "The CD looks good but I don't teach engineers: why isn't the same material available in different eBooks, e.g. one on multivariable calculus, one on ordinary differential equations, etc.?" The material in it is worth using for classes other than engineers. The eBook genre will grow: see the maplesoft web site for more titles.

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N is a Number
A Portrait of Paul Erdös

George Paul Csicsery
Springer VideoMath (DVD) 2005
ISBN: 3-540-22017-8

This documentary film is an inspiration and a delight.

I first met Paul Erdös when he visited Adelaide in 1972, and mark the real beginning of my research career from the time of that encounter. Our acquaintance was renewed with several subsequent visits. Hence this ‘final’ reacquaintance came as a great pleasure.

The film begins in Cambridge 1991, with Erdös being awarded an Honarary Doctorate at the age of 78 years. Then starting with his birth in Budapest in 1913, it retraces his long and eventful life through a skilfully edited sequence of interviews, and film clips of Erdös himself. I was interested in the appearance of British number theoreticians Hardy, Littlewood, Davenport and Cassels, an interview with local (Australian) identity Marta Sved, and mention of George Szekeres and Esther Klein (Szekeres).

Erdös was the most prolific mathematician who has ever lived, with an output of over 1300 papers. He was always interested in ‘elementary’ problems: problems which could often be understood by school students, although not usually easily solved. ‘How many?’ ‘How big?’ were often motivating questions. A number of these problems are outlined in a graphic and easily understood way within the presentation.

Erdös was also well known for his idiosyncrasies. For more than 50 years he travelled the world, exchanging board and lodging for a wealth of shared mathematical ideas and inspiration. He liked having people around, but often seemed to be lonely. He defined a mathematician as a machine that turns coffee into theorems. He often offered large sums of money for solutions to mathematical problems, although it is unclear where the money came from! I remember Erdös for his single-minded approach to mathematical problems: he seemed to be forever working on a problem. I was therefore interested to learn that he had much wider interests. Erdös believed that the best proofs are kept in ‘the book’. Thus new mathematics is not created; rather, mathematicians discover pages of ‘the book’. Erdös was well known for his incredible memory, his wry sense of humour, and his inexhaustible supply of historical anecdotes.

The DVD contains around an hour of film documentary, with the option of looking at certain ‘scenes’. There are also a further 20 minutes of interviews with Erdös on various topics. My review copy had a one-minute glitch near the end, but I assume this was a one-off.

In summary, this film is a wonderful eulogy to this gifted and amazing man. It will encourage, inspire and enthuse all who have an interest in mathematics, and fascinate those who have not. It is a great resource, and should be required viewing for all university mathematics staff and students.

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