

The behaviour of tsunamis

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1 Introduction

The behaviour of tsunamis is not easily understood. Readers' comments on a recently published paper [3] show that a mathematical analysis of a tsunami does not satisfactorily explain the matter to everyone. The following description of the mechanism of a tsunami is an attempt to clarify the situation.

A tsunami usually starts on deep ocean water as a result of a large-scale disturbance of the sea bed, such as an earthquake or an under water land slide. The surface of the ocean is raised (in most cases) by a small amount over a very large area, and this elevated water then flows out in all directions, starting a tsunami. It is the great area of the initial surface elevation which is responsible for a tsunami's enormous wave length, which may be many tens or even hundreds of kilometres. The period of the wave is very long, typically 15 minutes or more, and in mid-ocean the velocity of the wave profile may be many hundreds of kilometres per hour.

The height of a tsunami in mid-ocean is very small, typically less than one metre [2], making its passage there imperceptible to anyone on a ship. On reaching a shore, a tsunami may achieve great heights and deliver massive surges of water which can be very destructive. Between these forward surges the water at a shore line usually recedes for a long distance, leaving bare large areas of ocean floor that are normally under water

This paper investigates the behaviour of a tsunami, and explains how its mid-ocean character is transformed to produce massive surges of water at a shore.

2 Notation and definitions

Figure 1 shows a cross-sectional view of a wave on the surface of water, and some of the symbols used to describe it. The horizontal dimensions of the wave have been compressed below those relevant to a tsunami in mid-ocean and its vertical dimensions exaggerated. When investigating surface waves it is usual to refer them to Cartesian axes Ox , Oy located as shown.

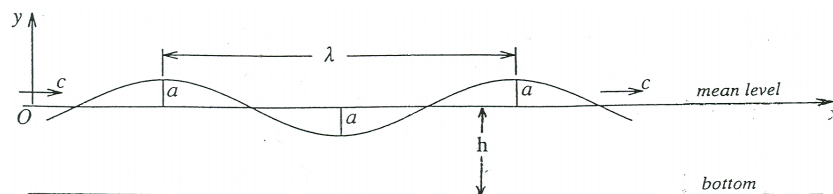


Figure 1. Progressive wave on the water surface

Let λ = wave length,
 a = amplitude of wave = half wave height,
 c = velocity of propagation of wave,
 h = depth of water.

A case in which the ratio h/a is very large will be called “deep water”; a case with h/a of moderate value will be called “shallow water”. (In mathematical wave theory the magnitude of h/λ is used for these definitions, but for a tsunami the huge value of λ makes this inappropriate.)

In Section 5 typical values will be ascribed to a and c , and will be used to predict the magnitude of the surges of water which would result when the tsunami reaches a shore.

3 The nature of wave motion

The mathematical theory of wave motion [4, 5] shows that it is only the wave profile depicted in Figure 1 which is continually moving forward with velocity c . The particles of water are not doing so, they are performing closed circuits around fixed points. In deep water (h/a large), the circuits are circular, the size of the circles decreasing as the depth increases, as shown in Figure 2. At a depth of half a wave length the radius of the circular motion is only about 4% of that at the surface.

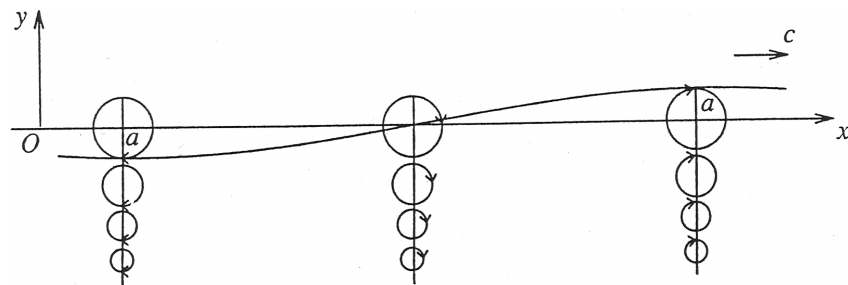


Figure 2. Circular paths of water particles in a wave on deep water

Figure 2 shows that at the water surface the radius of the circular paths of particles is the amplitude a of the wave, and that at a wave crest the water particles are moving forward in the direction of wave propagation, and at a trough the particles are moving backward.

In the case of shallow water (h/a not large), the mathematical theory of wave motion [4],[5] shows that the paths of particles are elliptical rather than circular, with the major (long) axes of the ellipses horizontal, as shown in Figure 3. The minor axes are vertical, their length at the water surface being equal to the height of the wave there. As the depth below the surface increases, the lengths of the major and minor axes decrease, as shown. At the ocean floor the ellipses degenerate into straight lines along which the water particles move back and forth as the wave passes.

Apart from the elliptical shapes of their orbits the behaviour of the water particles in the shallow water case is just the same as in the deep water case in Figure 2.

The wave in Figure 3 has been compressed horizontally to enable half a wave length to be shown, which would be hundreds of kilometres long in a typical case.

The movements of water particles depicted in Figures 2 and 3 are experienced by every particle in the neighbourhood of a wave. Acting together they constitute back and forth

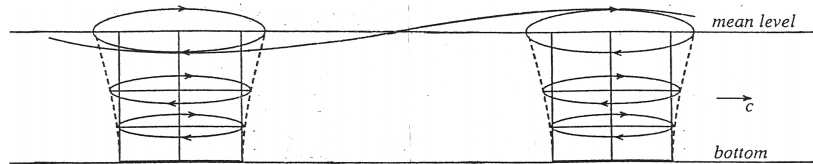


Figure 3. Elliptical paths of water particles in a wave on shallow water

movements of masses of water which are huge because of the great length of the waves of a tsunami. The magnitude of these water masses will be considered in the following sections.

4 The transport of water mass by a tsunami

The circular or elliptical movements of the water particles, which occur during passage of a wave, decrease in size with increasing depth, and their directions vary, as indicated by the arrows in Figure 4. Near a wave crest all particles have a forward component of velocity, and near a trough all have a backward velocity component.

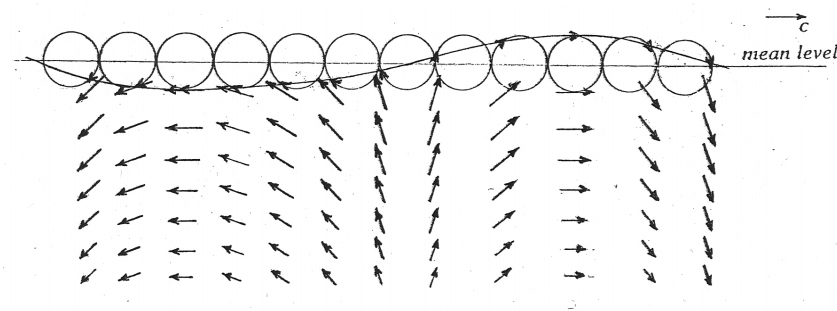


Figure 4. Directions of transport of water mass beneath a wave

Because of the great depth of water in mid-ocean, and the huge wave length of a tsunami (often hundreds of kilometres), the moving masses of water depicted in Figure 4 are enormous. They constitute huge surges of water, in the forward direction in the neighbourhood of a wave crest, and in the backward direction near a trough.

The wave passes any point in its path which is fixed in the ocean, and consequently at such a point there are successive surges of water in the forward and backward directions. The average horizontal velocity of the water in these surges is not the wave profile velocity c , it is the much smaller value associated with the circular or elliptical motions of the water particles. This is an important point, as it explains why the water surges of a tsunami at a shore occur at much smaller velocities than the profile velocity c .

5 Behaviour of a tsunami at a shore line

Far from the initial upheaval the motion is virtually two-dimensional, as depicted in Figure 4, and the rate of transport of mass is still huge. As a tsunami approaches a shore line and the water gets shallower, the circular movements of the water particles are elongated into ellipses as shown in Figure 3. Eventually the water becomes too shallow for the water

particles to complete their elliptical cycles, and the wave breaks and degenerates into an advancing wall of foaming water.

Water is virtually incompressible, which means that the huge magnitude of the mid-ocean back and forth surges depicted in Figure 4 remains unchanged as the wave progresses. This is the main factor responsible for the severe impact of some tsunamis on reaching a shore. The ocean floor slopes upward near a shore, and the forward surge of water can only be accommodated by an increased water surface level, adding to the familiar destructive impact of a tsunami.

The backward surges of water occurring between the forward ones have the same huge magnitude as the forward ones. Close to a shore their effect is to drain the water away from the shore, leaving large areas of the sea floor exposed. This can provide any local residents with a sign that a damaging tsunami wave will soon arrive, but if a forward surge is the first to reach a shore there will be no such warning of possible disaster.

When the surface of water is disturbed by a sudden shock like the dropping onto it of a small stone, a small train of waves called a “wave packet” is set in motion, spreading out away from the disturbance. It is well known that the leading wave in the packet continually dies away, and that a new wave continually appears at the rear of the packet [1]. This means that at all times the water of the leading wave is gradually changing between forward and backward surging states. Though there are no hard data on the matter, it is reasonable to believe that the same behaviour occurs in a tsunami. It is therefore not possible to predict whether a tsunami will be in its forward or its backward surging state when it arrives at a shore; it could be either, depending on the state of the wave and the location of the shore.

Waves travel more slowly as the water gets shallower, so if a wave front approaches a shore line at an angle the leading part of the front is slowed down more than the rest of it. This causes the wave front to swing around to become more nearly parallel to the shore, an effect which is known as wave refraction [1]. This is why waves including tsunamis breaking on a shore are composed of fronts which are parallel to the shore line, or nearly so.

The mathematical theory of wave motion [4, 5] shows that the rate R of transport of water volume (per unit length perpendicular to the plane Oxy of Figure 1 is given by

$$R = ac$$

Let us use this result to investigate the impact on a shore of a typical tsunami. The amplitude and velocity of propagation of a tsunami in mid-ocean could be

$$a = 0.4\text{metres}, \quad c = 700\text{km/hr} = 194\text{m/s}.$$

For such a wave, beneath each wave crest (per metre parallel to the shore line) the rate of forward transport of water would be

$$R = ac = 0.4 \times 194 = 78\text{m}^3/\text{s}.$$

At the shore line the height of the forward water surge could be about 10 metres in a typical situation, in which case its forward speed would be 7.8m/s, or 8m/s approximately. This is about as fast as most people can run, which explains why so many people are engulfed by an approaching tsunami. Local topographical features can affect the behaviour of waves approaching a shore line, including tsunamis. For example, refraction of waves approaching a headland may concentrate their impact [1], sometimes with devastating results.

6 Tsunami warning systems

In 1946 a tsunami overwhelmed the town of Hilo in the Hawaiian Islands, which led to a decision to develop a warning system for tsunamis in the Pacific Ocean [2]. One of the

systems developed depends on a centre in Honolulu receiving prompt reports from several Coast Survey stations in the Pacific which are equipped with seismographs. Each station can determine the direction to the epicentre of any submarine earthquake, and the intersection of these directions pinpoints the location of the epicentre. Ocean depths are known well enough to make possible the calculation of wave travel times from the epicentre to places which may be vulnerable to any resulting tsunami. Warnings can be issued to people in these places that a tsunami may occur at a particular time.

The Pacific tsunami warning system has proved very useful, reducing the loss of life in this region. Recent events have shown the desirability of installing a similar warning system in the Indian Ocean region.

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References

- [1] W. Bascom, *Ocean Waves*, Scientific American (August 1959).
- [2] J. Bernstein, *Tsunamis*, Scientific American (August 1954).
- [3] M.N. Brearley, *About tsunamis*, Math. Gaz. **89** (2005), 437–440.
- [4] L.M. Milne-Thomson, *Theoretical Hydrodynamics*, (Macmillan 1949).
- [5] A.S. Ramsey, *A Treatise on Hydromechanics*, Part II, (Bell 1960).

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