Math matters

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How "hard" are the hard sciences?

Mathematics, Physics and Chemistry are routinely referred to as the "hard sciences". Of course, I much prefer the terminology adopted by Batterham [1], who refers to them as the Fundamental Enabling Sciences, which better acknowledges their pivotal role in our society.

Nevertheless, the perception persists that these subjects are uncompromisingly "hard", and that's just how it has to be. By all accounts, this has resulted in a big turn-off for high-school students and those in the university sector, prompting the Australian Council of Deans of Science to remark that "Mathematics continues to be in difficulty ..." [2]. It is a well-worn cliché that those of us who love this "hard" subject must therefore necessarily live a monastic existence, with little time or use for normal social graces and friendships [3], must sport a type of Einstein hairdo and a rasping Julius Sumner Miller speaking voice.

I don't particularly want to go down the well-trodden and comfortable path of bemoaning the decline in Mathematics, the brain-drain and all the rest of it again here. While those issues do worry me, I suspect we may still have over-stated that side of things somewhat.

However, it has to be acknowledged that the perception that Mathematics (along with Physics and Chemistry) are "hard" is not exactly helping the cause of university Mathematics departments. In the Alice-in-Wonderland world of university budgets, departments are funded according to their student load, so that falling student numbers translates to fewer staff, which in turn means a reduced capacity to undertake research and teaching, and so on. The national interest seems not to figure in any of this. So if students get the idea that Mathematics is hard and therefore "un-cool", from night-time television shows or whatever, they are now free just to avoid the subject, which as a result struggles to survive. (There is something delightfully silly about all this; for example, can we expect a sudden increase in the national capacity to undertake research in forensic science in three years' time, as a result of the current popularity of the American television show "CSI"?).

So are Mathematics (and Physics and Chemistry) really "hard" sciences? I want to argue that they are not. Furthermore, I'd like to suggest that perhaps the best way of reversing the fortunes of these subjects is to convince people around us that these sciences are no harder than many other areas of human activity. In doing so, we may also need to change a few of our own perceptions along the way.

Mathematics is the language of technology, and during the course of its history, much of it was invented precisely for that purpose. Like all languages, it has its own notation, grammar and syntax. Learning these can occasionally be tedious, but no more so than learning the grammar of any other language. The symbols of mathematics can be off-putting and appear "hard" to the new student, but they are nowhere near as difficult to learn as to write in Japanese (for a Westerner such as myself). Like any other language, Mathematics has its own culture and heritage, and once the basic language has been learned, it is possible to express ideas within that culture with a remarkable simplicity and clarity.

At the University of Tasmania, Mathematics is combined with Physics in a single School. This has forced me to see the "hard" science debate from a perspective wider than just the Mathematical one. While we love to rehearse the arguments about decline and the brain-drain in Mathematics, I think it's fair to say that Physics has suffered far more in this respect than Mathematics has. I believe a key reason for this has been the entrenched and continuing view in many Physics departments that for Physics to be done properly, it has to be "hard" and unyielding, particularly so because it has a strong practical component. As a result, Physics departments all around the country have lost students and even whole service subjects, and are only now beginning to reverse the decline this has caused. So is Physics really "hard"? Yes, in some respects, but I suspect that learning to play the violin is harder. In Mathematics, we too had a similar view, but it seems we were able to intervene at an earlier stage than was the case for Physics; this may, however, have been more good luck than good management.

Why is it that Mathematics is seen as "hard"? We have all had the cocktail-party experience of announcing that we do mathematics for a living, only to watch our fellow party goers back away from us in discomfort, muttering something about "not being good at maths at school". Why does the mention of Mathematics elicit this reaction, in a way that other disciplines do not? It's hard to know for sure, but I suspect there are three contributing factors.

Teaching style

It often seems to me that Mathematics makes the mistake of teaching its concepts deductively rather than inductively. That is, a result will be introduced in its most general possible form, couched in a sea of impenetrable jargon and hedged about with a host of conditions and exclusion clauses. In this form it appears remote, inaccessible and unassailable, more like a religious icon than a scientific principle. The useful results of practical interest in almost every situation are then obtained as trivial special cases of this general statement. The poor student is too often left wondering where such an idea could possibly have come from, and who on Earth could ever have thought it up in the first place. The answer, of course, is that nobody did; rather it evolved over a period of time, in response to a mixture of practical technological questions as well as more abstract theoretical considerations.

It wasn't always like this, however. Take the case even of that most pure of pure mathematicians G.H. Hardy, who famously observed: "I have never done anything 'useful'."[4] In his treatise on divergent series [5], he explains concepts in language like this:

"Newton and Leibniz, the first mathematicians to use infinite series systematically, had little temptation to use divergent series (though Leibniz played with them occasionally). The temptation became greater as analysis widened, and it was soon found that they were useful, and that operations performed on them uncritically often led to important results which could be verified independently."

A little later in the same book, Hardy says: "Alternatively, we could, by a more daring calculation, deduce ...".

There is nothing pretentious or inaccessible about this. Instead, we get to see the precise thought patterns taking place, and we are invited to take an intellectual risk along with the writer. This is engaging stuff, and it's a pity that much more mathematics isn't written in this style.

Computer packages may help break down the deductive style of traditional Mathematics teaching somewhat, since they do allow a student to experiment with ideas and pictures. In this way, a more inductive learning style may be helped along a little, so that students get the chance to build up a general understanding of a concept from particular instances of it. (I am referring here specifically to *mathematical* computer packages, rather than to more general software for making lecture material accessible over the web. This latter seems to be much loved by university executives now far removed from daily interaction with students, but I remain to be convinced of its merits as a primary tool for teaching Mathematics).

Nevertheless, some changes to Mathematics curricula are probably still in order, and it would undoubtedly help even to add some background material about the arguments that were going on at the time the concept was invented (as Hardy did), and why this form of it seemed the best way to address the concerns of the time. A subject suddenly seems a lot less "hard" when there is a clear and engaging reason behind its actions. (To their credit, Physics courses often instill in the students a clear idea of why a concept was developed, and what question it was designed to answer).

Insularity

It often seems to me that Mathematics projects an air of being supremely disinterested in what the rest of Science is doing. It is not uncommon to see research papers that begin in the subjunctive tense, with a phrase something like: "Let X be a ..." followed by a lengthy string of abstruse adjectives and nouns. The next sentence will then be something like: "We prove the following theorem". It strikes me that there is a world of vital information missing from an introduction of that sort. Where is the initial discussion of why this result is of interest? How did such a question arise, and why are the conditions that surround the theorem of significance? Once the theorem has been duly proved, how does the result connect with what other people are doing, and what then is the next likely step? A result like this seems "hard", probably not so much because of the intellectual brilliance of its content (although no doubt its author would beg to differ on that point), but more because its very insularity makes it seem remote and forbidding.

Mathematicians are sometimes resentful of being seen as playing a mere support role to other more glamorous endeavours. I believe they are mistaken for thinking that way, if for no other reason that they underestimate the extent to which other Scientists genuinely recognize the worth of their skills and want access to them. A few years ago, I was required by my university to write a "strategic plan" for our School of Mathematics and Physics. Putting to one side my own reservations about documents of that sort, I tried to reflect the political realities of that time in the usual bureaucratic language designed for that purpose, and I wrote something to the effect that we would undertake a vigorous programme of research, in a variety of areas of modern Mathematics, "supported by fundamental research in pure mathematics...". To my surprise, a number of people objected to this language, essentially to the effect that their research stood on its own merits, and was not intended to support anyone. My own view is rather different to this, since I believe that Mathematics historically has done its best and most exciting work when it has collaborated actively with some external project. Tony Dooley [6] and Peter Taylor [7] have expressed it perfectly, with their phrase "the mysterious process between theory and applications".

Exclusivity

There are occasions when I think Mathematicians expend too much energy deciding who is in, and who is out. My own view is that anyone who loves Mathematics and who teaches it with passion and enthusiasm has earned the right to be considered fully a Mathematician. Yet I have heard it said in departmental tea-rooms and the like that a certain person is not really a Mathematician, because he or she "does not prove theorems". While I acknowledge the usefulness of the Theorem as a rhetorical construct for presenting an idea succinctly and efficiently, it is by no means the only way of expressing mathematical truth. It would be particularly sad if the work of an enthusiastic person were to be defined into irrelevance on so flimsy a basis as a mere rhetorical presentation style. Our subject deserves to be taken more seriously than that.

In a similar vein, there are areas of Mathematics where the residents seem to take delight in making hyper-fine distinctions between themselves and their close neighbours. To an outsider, it all seems very confusing. I remember once being in yet another Mathematics department tea-room, where the work of an (absent) third party was being discussed. Apparently this person had worked on topics such as Hopf algebra, Schur algebra, Hecke algebra, Weyl algebra, and then somebody observed sagely: "I guess he would consider himself to be an algebraist". I promptly burst out laughing, very much impressed at such sardonic humour, but I was immediately met by a room full of disapproving stares. Evidently the comment was meant in all seriousness, a fact which baffles me to this day.

I believe we often under-estimate the extent of Mathematical training and expertise possessed by some of our colleagues in other disciplines, too. This is why I feel some of our arguments about the decline of Mathematics and the brain drain are not wholly accurate. In a sense, it is an acknowledgement that Mathematics is considered too important just to leave to those of us in mathematics departments, although this can pose an extra concern for us in retaining our students. However, that is primarily a political question, having more to do with the way in which groups of people in a university are organized, than necessarily with the decline of Mathematics itself. During my three years on the ARC college of experts, for example, I was often asked about the "under-representation" of Mathematics on the panel. This came about because it was quickly seen that there were only two people on the panel who had a formal connection to a mathematics department, but it completely overlooked the very substantial Mathematical expertise of a number of the other members.

In this opinion piece, I've tried to argue that Mathematics in particular, and the physical sciences in general, are not really "hard" sciences, at least in the sense of being unusually difficult. In fact, the real power of Mathematics is that many of its central ideas possess an elegant and beautiful simplicity; this then gives them a universality that cuts across a wide variety of different subject and application areas. I believe that one of the things that holds Mathematics (and Physics) back is that some of the people who communicate it to others still allow themselves the luxury of supposing it to be fundamentally "hard", more so than other activities. Where it is manifestly not difficult, it is nevertheless unwittingly made to appear so by some of the means discussed above. Who knows what culture change we may be able to effect by a determination to convince people that, far from being pointlessly "hard", the real purpose of Mathematics is to bring order and simplicity to otherwise difficult situations?

References

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